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Multi-attribute target-based utilities and extensions of fuzzy measures

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Abstract

We introduce a formal description of the Target-Based Approach to utility theory for the case of $n > 1$ attributes and point out the connections with aggregation-based extensions of capacities. Our discussion provides economic interpretations of different concepts of the theory of fuzzy measures. In particular, we analyze the meaning of extensions of capacities based on n-dimensional copulas. The latter describes stochastic dependence for random vectors of interest in the problem. We also trace the connections between the case of $\{0, 1\}$-valued capacities and the analysis of "coherent" reliability systems.
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1. Introduction

Rich literature has been devoted in the last decade to the *Target-Based Approach* (TBA) to utility functions and economic decisions (see [4,5,8,9,34,35]). This literature is still growing, with main focus on applied aspects (see, for example, [2,37,38]).

Even from a theoretical point of view, however, some issues of interest demand further analysis. In this direction, the present paper will consider some aspects that emerge in the analysis of the multi-attribute case. Generally TBA can provide probabilistic interpretations of different notions of utility theory. Here we will in particular interpret in terms of stochastic dependence the differences among copula-based extensions of the same fuzzy measure.

In order to explain the basic concepts of the TBA it is, in any case, convenient to start by recalling the single-attribute case. Let $\Xi := \{X_\alpha\}_{\alpha \in A}$ be a family of real-valued random variables, that are distributed according to probability distribution functions F_α. Each element $X_\alpha \in \Xi$ is seen as a *prospect* or a *lottery* and a Decision Maker is expected to conveniently select one element out of Ξ (or, equivalently, $\alpha \in A$). Let $U : \mathbb{R} \to \mathbb{R}$ be a (non-decreasing)

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utility function, that describes the Decision Maker's attitude toward risk. Thus, according to the *Expected Utility Principle* (see [36]), the DM's choice is performed by maximizing the integral

$$\mathbb{E}\big[U(X_\alpha)\big] = \int_{\mathbb{R}} U(x)\,dF_\alpha(x).$$

In the Target-Based approach one in addition assumes U to be right-continuous and bounded so that, by means of normalization, it can be seen as a probability distribution function over the real line. This approach suggests looking at U as at the distribution function F_T of a random variable T. This variable will be considered a *target*, stochastically independent of all the prospects X_α. If T is a (real-valued) random variable stochastically independent of X_α in fact, one has

$$\mathbb{E}\big(F_T(X_\alpha)\big) = \int \mathbb{P}(T \le x) F_\alpha(dx) = \mathbb{P}(T \le X_\alpha),$$

and then, by setting $U = F_T$, the Expected Utility Principle prescribes a choice of $\alpha \in A$ which maximizes the probability $\mathbb{E}[U(X_\alpha)] = \mathbb{P}(T \le X_\alpha)$.

The conceptual organization and formalization of basic ideas have been proposed at the end of nineties of last century by Castagnoli, Li Calzi, and Bordley. Some arguments, that can be regarded nowadays as related with the origins of TBA, had been around however in the economic literature since a long time (see [4,8] and references therein).

After the publication of these papers, several developments appeared in the subsequent years concerning the appropriate way to generalize the TBA to the case of multi-attribute utility functions, see in particular [5,34,35]. As already mentioned such an approach, when applicable, offers probabilistic interpretations of notions of utility theory, and this is accomplished in terms of properties of the probability distribution of a random target. Such interpretations, in their turn, are easily understandable and practically useful. In particular, they can help a Decision Maker in the process of assessing her/his own utility function.

A natural extension of the concept of Target-Based utility from the case $n = 1$ to the case of $n > 1$ attributes is based on a specific principle of individual choice pointed out in [5]. In this paper, we formalize such a principle in terms of the concept of capacity and analyze a TBA multi-attribute utility as a pair (m, F) where m is a capacity over $N = \{1, \dots, n\}$ and F is an n-dimensional probability distribution function. For our purposes it is convenient to use the Sklar decomposition of F in terms of its one-dimensional margins and of its connecting copula. In such a frame, some aspects of aggregation functions and of copula-based extensions of capacities emerge in a straightforward way.

More precisely, the paper will present the following structure. In the next section, we will introduce the appropriate notation and detail the basic aspects of the multi-criteria Target-Based approach. Starting from the arguments presented in [5], we show how every Target-Based n-criteria utility is basically determined by a couple of objects: an n-dimensional probability distribution function and a fuzzy measure over $N := \{1, \dots, n\}$. This discussion will allow us to point out, in Section 3, that some of the results presented by Kolesárová et al. in [21] admit, in a completely direct way, probabilistic interpretations and applications in terms of the TBA. It will in particular turn out that n-dimensional copulas, that can be used for the extension of fuzzy measures, describe stochastic dependence among the components of random vectors relevant in the problem. Section 4 will be devoted to the special case of $\{0, 1\}$-valued capacities. We shall see how, under such a specific condition, our arguments are directly related to the field of reliability and of lattice polynomial functions. Some final remarks concerning the relations between the parameters of TBA utilities and economic attitudes of a Decision Maker will be presented in Section 5. The notation we used is motivated by our effort to set a bridge between the two different settings. The term "attributes", as used in the present paper, is substantially a synonymous of "criteria".

2. Multi-attribute target-based utilities

In this section we deal with the TBA form of utility functions with $n > 1$ attributes. As recalled in the introduction, in the single-attribute case, $n = 1$, a TBA utility is essentially a non-decreasing, right-continuous, bounded function that, after suitable normalization, is regarded as the distribution function of a scalar random variable T with the meaning of a target. Actually even more general, non-necessarily increasing, "utilities" can be considered in the TBA

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