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Fuzzy Bi-cooperative games in multilinear extension form

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Abstract

In this paper, we introduce the notion of a fuzzy Bi-cooperative game in multilinear extension form. An LG value as a possible solution concept is obtained using standard fuzzy game theoretic axioms.
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Keywords: Fuzzy sets; Bi-cooperative games; Bi-coalitions; LG value

1. Introduction

In this paper, we propose a Bi-cooperative game with fuzzy bi-coalitions in multilinear extension form. Owen's [12] multilinear extension of a game is a very important tool in game theory particularly for computing the Shapley like solutions for large games. Moreover, it serves as a tool to characterize many related concepts, a remarkable one being identification of its linkage with the Choquet integral [7]. Following Owen, we integrate the multilinear extension over a simplex to construct a new class of Bi-cooperative games with fuzzy bi-coalitions. Meng and Zhang [10] defined a fuzzy cooperative game in multilinear extension form and obtained the characterization of the corresponding Shapley value.

Theory of Cooperative games since its inception by von Neumann and Morgenstern [13] has been instrumental in building decision models where a group of people (players) indulge in a joint endeavur with the single motive to gain more than what they would generate individually. However, it is found to be insensitive to the situations where a second group of players opposes the formation of the former group and the rest of the players remain indifferent. This idea extends the notion of a coalition to a bi-coalition: a pair of mutually exclusive coalitions of which the former coalition comprises of the positive contributors and the later coalition comprises of the negative contributors. Bipolarity of this kind was initially modeled in ternary voting games by Felsenthal and Machover [4]. Bilbao et al. [2] proposed a more general framework and called the corresponding games the Bi-cooperative games. They defined

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the order related to the bipolar monotonicity among the bi-coalitions to make the corresponding class a distributive lattice. Hsiao and Raghavan [8] introduced the notion of multi-choice games where players have contributions to coalitions at finitely many distinct levels. Here a partition of the player set exists with each member being labeled from zero (no participation) to some fixed number (highest level of participation). In this sense the Bi-cooperative games are a particular class of (or isomorphic to) the multi-choice games with three levels of participations. However in [9], Labreuche and Grabisch observed that the class of Bi-cooperative games should differ from their multi-choice counterparts because of the bipolarity they adhere. They have shown that by considering the product order instead of the one implied by monotonicity adopted by Bilbao et al. [1], one can distinguish them from the multi-choice games. Moreover it is interesting to note that under this product order, the class of bi-coalitions becomes an inf-semilattice.

A solution to a Bi-cooperative game is a payoff vector that satisfies some pre-imposed rationality conditions. The ith component of the vector represents the payoff to the ith player after she chooses her role in the game. Bilbao et al. [1] obtained the Shapley value for the class of Bi-cooperative games. Labreuche and Grabisch [9] proposed an alternative solution concept which we call here the LG value. These two rules differ by the underlying lattice structures imposed on the set of bi-coalitions. The LG value provides a more natural justification to the notion of bipolarity that distinguishes it from the multi-choice games.

In crisp Bi-cooperative game the membership of players (rates of participation) is assessed in binary terms (i.e., 1 for participation in the game, and 0 for nonparticipation). By contrast, fuzzy set theory permits the gradual assessment of the memberships of players (rates of participation) in a game. When in a coalition, players are participating partially with some membership degrees in the interval [0, 1], we call it a fuzzy coalition. In the similar fashion, when players participate partially in a bi-coalition, we can call it a fuzzy bi-coalition. The notion of a Bi-cooperative game with fuzzy bi-coalitions and its intuitive justifications were proposed by Borkotokey and Sarmah [3] where they obtained a set of axioms for the characterizations of the LG value [9] under fuzzy environment. The class of fuzzy Bi-cooperative games in Choquet integral type was introduced and finally the corresponding LG value for this class was obtained. In the present paper, we obtain an LG value for the class of fuzzy Bi-cooperative games in multilinear extension form. Possible relationships with the existing models are explored.

The rest of the paper is organized as follows. Section 2, presents the notion of Bi-cooperative games and corresponding solution concepts in both crisp and fuzzy environments. In Section 3, we introduce the notion of a fuzzy Bi-cooperative game in multilinear extension form. An LG value for this class has been proposed and shown to satisfy the LG axioms given in [3]. Section 4 includes the concluding remarks.

2. Bi-cooperative games in crisp and fuzzy settings

In this section, we present the basic definitions and results of Bi-cooperative games with both crisp and fuzzy coalitions and also define the LG value as a suitable solution concept. To a large extent, this section is based on Bilbao et al. [1,2], Labreuche and Grabisch [9] and Borkotokey and Sarmah [3]. Throughout the paper $N = \{1, 2, 3, \dots, n\}$ denotes the players' set and $\mathcal{Q}(N) = \{(S, T) \mid S, T \in N \text{ and } S \cap T = \emptyset\}$, the set of all bi-coalitions of N. Further, we assume that the members of $(S, T) \in \mathcal{Q}(N)$ exhibit bipolarity through their contributions to S or T. By what is called abuse of notations we alternatively use i for the singleton set $\{i\}$. Denote by small letters the cardinalities of sets, e.g., s for S etc.

2.1. Bi-cooperative games with crisp bi-coalitions and the LG value

A Bi-cooperative game is a pair (N, v) of which N is the players' set and $v : \mathcal{Q}(N) \to \mathbb{R}$, a real valued function such that $v(\emptyset, \emptyset) = 0$. In what follows, we formally present the two important ordering relations defined on $\mathcal{Q}(N)$ and their possible implications, see [2,9]. The first relation $\sqsubseteq_1$ defined by Bilbao et al. [2] is implied by monotonicity, i.e., for $(S, T), (S', T') \in \mathcal{Q}(N)$, $(S, T) \sqsubseteq_1 (S', T')$ iff $S \subseteq S'$ and $T' \subseteq T$. This makes the two elements $(\emptyset, N)$ and $(N, \emptyset)$ as the bottom and top elements of $\mathcal{Q}(N)$ and $\mathcal{Q}(N)$ becomes a distributive lattice. A second relation: the product order $\sqsubseteq_2$ given in [9] is defined as follows. For $(S, T), (S', T') \in \mathcal{Q}(N)$, $(S, T) \sqsubseteq_2 (S', T')$ iff $S \subseteq S'$ and $T \subseteq T'$ so that $(\emptyset, \emptyset)$ becomes the bottom element and all $(S, N \setminus S)$, $S \subseteq N$, the maximal elements. Under this ordering relation, $\mathcal{Q}(N)$ is an inf-semilattice. Note that this relation is in cognition with the notion of Bi-cooperative games as it incorporates bipolarity, a crucial concept for such games. Therefore, in this paper, we shall adopt this ordering relation and denote it simply without a suffix i.e., by $\sqsubseteq$.

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