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Multidimensional coincidence point results for compatible mappings in partially ordered fuzzy metric spaces

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Abstract

In recent times, *coupled*, *tripled* and *quadruple* fixed point theorems have been intensively studied by many authors in the context of partially ordered complete metric spaces using different contractivity conditions. Roldán et al. showed a unified version of these results for nonlinear mappings in any number of variables (which were not necessarily permuted or ordered) introducing the notion of *multidimensional coincidence point*. Very recently, Choudhury et al. proved coupled coincidence point results in the context of fuzzy metric spaces in the sense of George and Veeramani. In this paper, using the idea of coincidence point for nonlinear mappings in any number of variables, we study a fuzzy contractivity condition to ensure the existence of coincidence points in the framework of fuzzy metric spaces provided with Hadžić type t-norms. Then, we present an illustrative example in which our methodology leads to the existence of coincidence points but previous theorems cannot be applied.

Keywords: Fuzzy metric space; Coincidence point; Fixed point; Contraction; Mixed monotone property; Partially ordered set; Compatible mappings; Fuzzy topology; Hadžić type t-norm

1. Introduction

Taking into account its interesting applications, Fixed Point Theory has received considerable attention through the last ninety years in many different ways. One of the newest branches of this theory is devoted to the study of *coupled fixed point*. The notion of coupled fixed point was introduced by Guo and Lakshmikantham [12] in 1987. In a recent paper, Gnana-Bhaskar and Lakshmikantham [3] introduced the concept of *mixed monotone property* for contractive operators in two variables over a partially ordered metric space, and then established some coupled fixed point theorems. After that, many results appeared on this field in different contexts (see e.g. [9,14,21,27,29]). Later, Berinde and Borcut [2] introduced the concept of *tripled fixed point* and proved tripled fixed point theorems using mixed monotone mappings (see also [1,5,25]). Quadruple fixed point theorems also appeared (see [16–20]) because

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there were two open problems: how to generalize these results to higher dimension and how the variables could be permuted.

To answer to both questions, Roldán et al. [23] proposed the notion of *coincidence point* between mappings in any number of variables and showed some existence and uniqueness theorems that extended the mentioned previous results for this kind of nonlinear mappings, not necessarily permuted or ordered, in the framework of partially ordered complete metric spaces, using a weaker contractivity condition, that also generalized other work by Berzig and Samet [4].

Very recently, Choudhury et al. [6] established coupled coincidence point and coupled fixed point results in the context of fuzzy metric spaces using compatible mappings that were not necessarily commuting, and particularized their results to metric spaces. In this paper, we present an existence result of coincidence points between nonlinear mappings in any number of variables on fuzzy metric spaces. Our results not only extend the above mentioned ones, but they generalize, clarify and unify several classical and very recent related results in literature in the setting of metric spaces (see [2–5,7,19,21,23]).

This paper is organized as follows. In Section 2, some notations and preliminary results are presented and, in Section 3, the main results are established. We also give an illustrative example in which our methodology leads to the existence of coincidence points but previous theorems cannot be applied.

2. Preliminaries

Let n be a positive integer and let $\Lambda_n = \{1, 2, \dots, n\}$. Henceforth, X will denote a non-empty set and X^n will denote the product space $X \times X \times \overset{n}{\cdots} \times X$. We represent the identity mapping on X as I_X. Throughout this manuscript, m and p will denote non-negative integers, t will be a positive real number and $i, j, s \in \{1, 2, \dots, n\}$. Unless otherwise stated, "for all m" will mean "for all $m \geqslant 0$" "for all t" will mean "for all $t > 0$" and "for all i" will mean "for all $i \in \{1, 2, \dots, n\}$". Let denote $\mathbb{R}^+ = (0, \infty)$ and $\mathbb{I} = [0, 1]$. In the sequel, let $F : X^n \to X$ and $g : X \to X$ be two mappings. For brevity, $g(x)$ will be denoted by gx.

2.1. A partial order on X^n

Henceforth, fix a partition $\{A, B\}$ of $\Lambda_n = \{1, 2, \dots, n\}$, that is, $A \cup B = \Lambda_n$ and $A \cap B = \emptyset$. We will denote:

$$\Omega_{A,B} := \{\sigma : \Lambda_n \to \Lambda_n\colon \sigma(A) \subseteq A \text{ and } \sigma(B) \subseteq B\}, \quad \text{and}$$
$$\Omega'_{A,B} := \{\sigma : \Lambda_n \to \Lambda_n\colon \sigma(A) \subseteq B \text{ and } \sigma(B) \subseteq A\}.$$

If $\preccurlyeq$ is a partial order on X (i.e., $(X, \preccurlyeq)$ is a partially ordered set), $x, y \in X$ and $i \in \Lambda_n$, we will use the following notation:

$$x \preccurlyeq_i y \quad \Leftrightarrow \quad \begin{cases} x \preccurlyeq y, & \text{if } i \in A, \\ x \succcurlyeq y, & \text{if } i \in B. \end{cases}$$

Consider on the product space X^n the following partial order: for $(x_1, x_2, \dots, x_n), (y_1, y_2, \dots, y_n) \in X^n$,

$$(x_1, x_2, \dots, x_n) \leqslant (y_1, y_2, \dots, y_n) \quad \Leftrightarrow \quad x_i \preccurlyeq_i y_i, \quad \text{for all } i.$$

We say that two points $(x_1, x_2, \dots, x_n)$ and $(y_1, y_2, \dots, y_n)$ are *comparable* if $(x_1, x_2, \dots, x_n) \leqslant (y_1, y_2, \dots, y_n)$ or $(x_1, x_2, \dots, x_n) \geqslant (y_1, y_2, \dots, y_n)$.

2.2. Mixed monotone mappings and coincidence points

Let $F : X^n \to X$ and $g : X \to X$ be two mappings.

Definition 1. (See [23].) We say that F and g are *commuting* if $gF(x_1, \dots, x_n) = F(gx_1, \dots, gx_n)$ for all $x_1, \dots, x_n \in X$.