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Fuzzy Sets and Systems 266 (2015) 131–143

FUZZY sets and systems

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A comparison of fuzzy regression methods for the estimation of the implied volatility smile function

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Received 24 December 2013; received in revised form 10 October 2014; accepted 14 November 2014

Available online 25 November 2014

Abstract

The information content of option prices on the underlying asset has a special importance in finance. In particular, with the use of option implied trees, market participants may price other derivatives, estimate and forecast volatility (see e.g. the volatility index VIX), or higher moments of the underlying asset distribution. A crucial input of option implied trees is the estimation of the smile (implied volatility as a function of the strike price), which boils down to fitting a function to a limited number of existing knots. However, standard techniques require a one-to-one mapping between volatility and strike price, which is not met in the reality of financial markets, where, to a given strike price, two different implied volatilities are usually associated (coming from different types of options: call and put).

In this paper we compare the widely used methodology of discarding some implied volatilities and interpolating the remaining knots with cubic splines, to a fuzzy regression approach which does not require an a-priori choice of implied volatilities. To this end, we first extend some linear fuzzy regression methods to a polynomial form and we apply them to the financial problem. The fuzzy regression methods used range from the possibilistic regression method of Tanaka et al. [28], to the least squares fuzzy regression method of Savic and Pedrycz [27] and to the hybrid method of Ishibuchi and Nii [11].

Keywords: Fuzzy statistics and data analysis; Finance; Fuzzy regression methods; Non-linear programming; Implied volatility

1. Introduction

The information content of option prices on the underlying asset has a special importance in finance. In particular, with the use of option implied trees, market participants may price other derivatives, estimate and forecast volatility (see e.g. the volatility index VIX), or higher moments of the underlying asset distribution. An option gives the holder the right to buy (call option) or to sell (put option) a financial instrument (the underlying asset) for a pre-specified

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http://dx.doi.org/10.1016/j.fss.2014.11.015

price (strike price), on a given date (expiry date). Option prices are quoted in the market for a discrete number (e.g. 15 in the Italian market) of different strike prices K, usually equally spaced, ranging from $K_{\min}$ (the minimum quoted strike price) to $K_{\max}$ (the maximum quoted strike price). An option is said to be at-the-money, out-of-the-money or in-the-money, if it generates a zero, negative, positive payoff respectively, if exercised immediately.

A crucial input of option implied trees is the estimation of the smile (implied volatility as a function of the strike price), which boils down to fitting a function to a limited number of existing knots (pairs of strike price and implied volatility). The main issue with the use of option prices is the generation of the missing prices for strikes that are not quoted, but are necessary in order to derive option implied trees or volatility forecasts. The way in which implied volatility varies with strike price is referred to as the "smile" (or smirk) effect, since depending on the market under scrutiny, it can be depicted with a smile (if implied volatility is higher at the edges of the strike price interval than it is in the middle) or a smirk (if implied volatility is higher for low strike prices than it is for high strike prices).

The no-arbitrage argument would imply that it is indifferent to obtain an implied volatility from a call or a put with the same strike price. Empirically, due to market frictions and the impossibility to perfectly replicate every claim, the two implied volatilities are different. Therefore, it is market practice to keep the implied volatility of put options for strikes below the current value of the underlying asset and the one of call options for strikes above (those options are called out-of-the-money, since if exercised they would deliver no positive payoff). The latter market practice is based on the observation that the options retained are the most exchanged and thus the most informative. For the at-the-money strikes (the one right below and right above the underlying asset value), an average of call and put implied volatilities is used.

The aim of this paper is to investigate the potential of fuzzy regression for the estimation of the smile. With fuzzy regression we should be able to use all the information of both call and put options, without having to make an a priori choice. Given that the majority of the papers in the literature concentrates on fuzzy linear regression and there is no ready-to-use model which can be adapted to our application, as a first step, we extend three of the most used linear regression methods to a polynomial form. The methods range from the possibilistic regression method of Tanaka et al. [28], to the least squares fuzzy regression method of Savic and Pedrycz [27] and to the hybrid method of Ishibuchi and Nii [11]. Second, the usefulness of fuzzy regression is evaluated by constructing an option implied tree with the estimated smile function and assessing the accuracy of the tree in pricing the same options used for its construction. Third, to leave nothing in doubt, we also assess the usefulness of the fuzzy regression methods in forecasting the real moments of the distribution (variance, skewness and kurtosis).

The results of the paper are particularly useful in at least two aspects. First, the extension to a polynomial form of the fuzzy regression methods may challenge the application to other interesting problems in finance and other disciplines, characterized by a non-linear relationship between dependent and independent variable. Second, at the practitioner level, we show that indeed, fuzzy regression could provide important improvements over standard techniques.

To the best of our knowledge, this is the first attempt of using fuzzy regression methods to estimate the smile function. A different approach to embed the conflicting information coming from call and put prices in an implied binomial tree is the one of Muzzioli and Torricelli [21,23] and Moriggia et al. [16], where two different implied trees (derived from two smile functions, one estimated using only call prices, the other using only put prices) have been derived and merged in a binomial tree with interval values for the stock price at each node. Although the problem of estimating the smile function (volatility as a function of strike price) has not been tackled with other fuzzy methods, various contributions in the literature address the problem of modelling (constant) volatility as a fuzzy quantity in a standard Black–Scholes [1] or Cox–Ross–Rubinstein [6] binomial model. In pricing European style options (which can be exercised only at the maturity date), we recall the contributions of Yoshida [33], Wu [32], Chrysafis and Papadopoulos [5], Liu and Li [15] in continuous time and Muzzioli and Torricelli [22] in discrete time; in pricing American style options (which can be exercised at any time before the maturity date) the work by Yoshida [34] and Muzzioli and Reynaerts [24]. Lastly, we recall the approach of Capotorti and Figà Talamanca [2] to estimate a membership function for fuzzy volatility, and the one of Thavaneswaran et al. [30] to estimate a fuzzy volatility autoregressive model.

The paper proceeds as follows. In Section 2 we provide a brief introduction to fuzzy regression methods. In Section 3 we recall the linear fuzzy regression methods of Tanaka et al. [28], Savic and Pedrycz [27] and Ishibuchi and Nii [11] and in Section 4 we extend them to the polynomial case. In Section 5 we present a simple example of estimation of the smile function with the different methods. Section 6 presents the data set, the methodology and the results. The last section concludes and provides some hints for future research.