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Robust stabilization design of nonlinear stochastic partial differential systems: Fuzzy approach

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Abstract

In this paper, we address the robust stabilization design problem for nonlinear stochastic partial differential systems (NSPDSs) in the presence of random external disturbances and measurement noise in the spatio-temporal domain. We propose a fuzzy stochastic partial differential system to approximate the NSPDSs based on fuzzy interpolation approach. Next, we develop a fuzzy stochastic spatial state space model to represent the fuzzy stochastic partial differential system by employing a semi-discretization finite difference scheme. Based on this model, we propose a robust fuzzy estimator-based stabilization controller to stabilize the NSPDSs. Furthermore, robust stochastic H_∞ stabilization design is suggested to attenuate the effects of random external disturbances and measurement noise in the spatio-temporal domain from the area energy point of view. The LMI technique is applied to solve the issue of control gains and estimator gains of the controller by using a systematic control design procedure. Finally, a simulation is presented as an example to illustrate the design procedure and to confirm the performance of the proposed robust fuzzy estimator-based stabilization design for the NSPDSs.

Keywords: Nonlinear stochastic partial differential systems; Partial differential equations; Finite difference scheme; Semi-discretization; Robust stabilization; Stochastic H_∞ stabilization; Fuzzy approach; Spatio-temporal random noises

1. Introduction

In recent years, a great deal of concern has been raised regarding the study of stochastic partial differential equations (SPDEs) [1–12]. Many phenomena in science and engineering have been modeled by deterministic partial differential equations (PDEs). Examples include mechanical systems related to heat flows, fluid flows, elastic waves, flexible structures, chemical engineering [13], biology, population dynamics, neurophysiology, biodynamics [14,15] etc. Since most phenomena possess some uncertainty due to the existence of different stochastic perturbations, SPDEs should be used to accurately represent their behaviors [1,2]. Some examples are turbulent flow in fluid dynamics [16], diffusion in random media, and molecule signal transduction in biology. In chemical engineering, many chemical processes are characterized by the presence of spatial variations and time delays [6,13]. In biology [14,15], mathematical tools can help provide systematic analysis, e.g. of the stability or robustness of biological systems. For example, the nonlinear

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partial differential Hodgkin–Huxley model has been applied to model signal transmission in a nervous system [14]. Here we address the study of nonlinear stochastic partial differential systems (NSPDSs).

In the past decade, control design problems for deterministic partial differential systems have been thoroughly investigated. The stabilization design problem of linear partial differential systems (LPDSs) has been particularly widely studied and published [17]. These LPDSs can be transformed into equivalent infinite-dimensional ordinary differential systems. According to the separation of eigenvalues, the infinite-dimensional ordinary differential system can be separated into a finite-dimensional slow mode and an infinite-dimensional fast mode. A finite-dimensional controller based on the finite-dimensional mode can be designed to stabilize the LPDS. For the reference tracking problem of LPDSs, Byrnes et al. proposed an extended output regulation method to control LPDS to track a reference model [18]. The stabilization design problem of nonlinear partial differential systems (NPDSs) is more complex than the control design of the LPDSs. Based on Galerkin's method, controller design schemes have been proposed to stabilize the NPDS by employing a residual model filter [19] or an inertial manifold model [20]. Due to the properties of nonlinearity and infinite dimension, the infinite-dimensional nonlinear ODE in [19] and [20] is difficult to obtain for the design of the corresponding controller. However, the best approach to the design of the corresponding controller is still unclear. In order to remedy this design difficulty, some researchers have provided systematic approaches based on fuzzy theory to solve the NPDSs stabilization design problem [21–25].

The fuzzy approach, which uses several local linear models to interpolate a nonlinear system, has been widely applied in the past twenty years to analyze various nonlinear systems which are described by ODE [26–32]. The fuzzy approach was recently applied to the domain of PDEs, resulting in the proposal of a new PDE solution technique using an adaptive fuzzy algorithm [33]. On the other hand, a fuzzy control method was also proposed to stabilize a class of NPDS based on the infinite-dimensional nonlinear ODE represented by Galerkin's method [21,22]. In [23], a new T-S fuzzy state space model using the fuzzy interpolation approach and Galerkin's method was proposed to approximate the NPDS. The advantage of the fuzzy state space model is that it avoids the necessity of obtaining a complex nonlinear ODE. As a result, the effects of the spillover on stabilization due to control and observation can be attenuated by the proposed robust stabilization design in [23]. A robust H_∞ stabilization scheme was developed to attenuate the effects of modeling errors, external disturbances, and measurement noises. In [34], the robust H_∞ reference tracking control problem for NPDSs was solved using the fuzzy approach and the finite difference method. A robust fuzzy observer-based tracking control scheme was designed to attenuate the effects of approximation errors, truncation errors, external disturbances, and measurement noises due to the H_∞ reference tracking performance.

Recent investigations of stochastic partial differential equations have focused on stochastic differential equation in infinite dimension [35]. In [36], the stochastic stability properties of linear stochastic second-order partial differential equations were introduced. The stability of NSPDSs was analyzed based on the functional Itô formula, the Lyapunov and related functions [5]. The concept of vector Lyapunov-like functional technique coupled with partial differential inequalities is described to investigate various types of stability related to NSPDSs using the comparison principle [4]. Moreover, various stability problems of NSPDSs have been presented in textbooks [1,2]. In the other words, numerical solutions for SPDEs have been studied, employing the finite difference method [3,7], the method of lines (semi-discretization) [8,9] and Taylor expansions [37]. However, few results have been presented for the control design of the SPDSs due to complex infinite-dimensional stochasticity. A predictive control model for NSPDSs with state-independent noise has been developed using Galerkin's method [6]. Since NSPDSs are the most complicated control systems, their robust stabilization design is a challenging control problem, especially in the case of random noise. We shall, therefore, discuss robust stochastic stabilization problems for NSPDSs with random external disturbances and measurement noises in the spatio-temporal domain.

In this paper, robust stochastic stabilization control design of NSPDSs is developed by employing the Itô formula, fuzzy interpolation, and the method of lines. First, the NSPDS is approximated using a so-called fuzzy stochastic partial differential system (FSPDS) by interpolating several LSPDSs via membership functions with some approximation errors. The FSPDS can then be represented by a finite-dimensional fuzzy stochastic spatial state space model based on the Kronecker product and the method of lines (semi-discretization finite difference method). Unlike using the infinite-dimensional ODE system to represent the PDE system [19,20,33], for the convenience of control design, the partial differential operator could be approximated by using a finite difference operator. The convergence of the method of lines was previously proven in [3,7–9,38]. This fuzzy stochastic spatial state space model can therefore