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Progressively better estimates of the domain of attraction for nonlinear systems via piecewise Takagi–Sugeno models: Stability and stabilization issues ☆

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Abstract

This paper introduces a novel approach for stability analysis and controller design of nonlinear models, both continuous- and discrete-time, based on an exact piecewise Takagi–Sugeno representation which generalizes the sector nonlinearity methodology. The motivation behind this work lies on the fact that piecewise-induced Takagi–Sugeno representations have smaller variations than single ones covering the whole modeling area, thus increasing the chances of finding piecewise solutions for stability analysis or controller synthesis. Moreover, this relaxation is shown to produce progressively better estimations of the domain of attraction via easy-to-implement algorithms that demand much less computational effort than other recent approaches in the literature. Examples are provided to highlight the advantages of the contributions over the existing methods.

1. Introduction

Originally intended to relate rule-based fuzzy representations with traditional control design, Takagi–Sugeno (TS) models first appeared in [1]; they were defined as a nonlinear convex blending of linear models where convex functions emerged from linguistic rules and linear models arose from linearization [2]: they were therefore *approximate*. Later on, the sector nonlinearity methodology allowed a nonlinear model to be *exactly* rewritten as a TS one in a compact set $\mathcal{C}$ of the state space, by capturing the model nonlinearities in membership functions (MFs) which held the convex-sum property [3]: this work adopts this approach. Thus, when exactly modeling a nonlinear system (a) a TS model is no longer an approximation, but an exact algebraic rewriting of the original one, regardless of where the state lies, (b) the MFs h_i are no longer state-independent entities belonging to a standard simplex $\Gamma = \left\{h_i \in \mathbb{R} : 0 \le h_i,\ \sum_i h_i = 1\right\}$,

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but *state-dependent* functions whose usefulness (i.e., whose convexity) is strictly limited to the modeling area $\mathcal{C}$. This consideration is important since most of the TS-based results for stability analysis and controller design disregard their inherent locality, perhaps due to the fact that their final conditions do not take into account the MFs and look therefore alike whether they come from an exact or an approximate approach, i.e., they are linear matrix inequalities (LMIs) [4,5] whose feasibility can be efficiently tested via commercially available software [6].

Stability and stabilization conditions in the original TS-LMI framework are only sufficient [7,8]. One source of conservativeness is the way the MFs are removed from convex nested sums in order to obtain LMI conditions, which is called the co-positivity problem and has been given several answers [9–12], all of which disregard any state-dependence of the MFs. Another source of pessimism is the sort of Lyapunov function involved, which instead of quadratic can be fuzzy (reproducing the convex structure of the TS model) [13–18], or piecewise (for TS models where not all their linear components are simultaneously activated, thus inducing state-space partitions) [19–22]: the former is obliged to deal with the time-derivative of the MFs for continuous-time models, the latter is inapplicable for exact TS models where all the MFs are simultaneously activated. The use of more general classes of convex models has been also explored as a way to relax the aforementioned conservativeness: descriptor models [23,24] face the same problems of Lyapunov function choice; polynomial ones [25,26] may lead to better results at a great cost in theoretical complexity, numerical burden, and limited applicability.

This paper considers that there is a compromise between complexity and quality of results that may have escaped from the picture above when stability and stabilization of nonlinear systems is investigated via TS models. To see this, consider a nonlinear system whose *exact* TS model is valid in $\mathcal{C}$. Results based on a piecewise Lyapunov function $V_p(x)$ (PWLF) associated to this system are valid for the largest region $\mathcal{D} = \{x : V_p(x) \leq \gamma\}$ in $\mathcal{C}$, while those based on a fuzzy Lyapunov function $V_n(x)$ (FLF, also known as non-quadratic) are valid for the largest region $\mathcal{D} = \{x : V_n(x) \leq \gamma\}$ in $\mathcal{C}$ as long as a third set of bounds on the time-derivatives of the MFs h_i hold: $\mathcal{S} = \{x : |\dot{h}_i(x)| \leq \phi_i\}$ [27]. Therefore, piecewise approaches may lead to larger feasible regions.

With this reasoning in mind, this paper has the following objectives:

1. *Modeling*: Since an exact representation should be available for analyzing nonlinear systems via TS models, this implies the use of the sector nonlinearity approach, but the TS models thus produced have all its linear consequents simultaneously activated, which precludes them from being treated under the piecewise approach. Efforts to overcome this problem have been made in [28,29] and [30] by mixing PWLFs with FLFs: they have produced very involved conditions where the continuity problem has not been addressed, except in [30] where the continuity is treated as in [19]. In [31] a generalization of the sector nonlinearity approach has been provided; it produces an exact piecewise TS (PWTS) representation of a nonlinear model: here, we pursue this investigation.
2. *Stability analysis:* Extend the methodology in [19] to exact PWTS models for stability analysis of both continuous- and discrete-time nonlinear systems showing (a) how an increasing number of partitions leads to progressively better feasibility sets which advantageously compare – both numerically and qualitatively – with other recent approaches such as [32–34] and (b) how the domain of attraction (DA) of an equilibrium point of a nonlinear system can be estimated with an arbitrary degree of accuracy via PWTS representations and an adequate algorithm, also performing better over recent similar approaches such as [26,27,35].
3. *Stabilization (controller design):* Extend the methodologies in [21,36] to (a) determine a piecewise control law $u(t)$ for nonlinear models via PWTS representations such that the closed-loop equilibrium point is locally asymptotically stable, both for continuous- and discrete-time systems, and (b) get progressively better invariant subsets of the DA that fit into the modeling region $\mathcal{C}$ improving over recent results concerned with the closed-loop DA [27,37]. Take into account that belonging to the modeling region $\mathcal{C}$ is an imposed condition in this sort of controller design, since the system thus modeled has physical constraints precisely reflected in the states belonging to $\mathcal{C}$, whereas for stability of an autonomous system the DA may easily go beyond the modeling area.

This article is organized in four more sections. Section 2 provides some notation, properties, and a generalization of the sector nonlinearity approach that allows a PWTS representation to be obtained from a nonlinear model on a compact set of its state space. Based on this representation, stability analysis is performed in Section 3 for continuous- as well as discrete-time systems via PWTS models to the extent of providing progressively better estimations of the DA of the equilibrium point. Section 4 considers the stabilization of nonlinear systems via PWTS representations; the algorithm for estimation of the DA is adapted to controller design. Illustrative examples and comparisons are