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Short Communication

Generalized uniform fuzzy partition: The solution to Holčapek's open problem

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Abstract

In this note, we propose a solution to an open problem, which was presented by Mesiar and Stupňanová. We give two counterexamples to show that the hypothesis is not sufficient to get the result and we propose that the sufficient condition of the hypothesis is held by modifying the symmetry condition.

Keywords: Uniform fuzzy partition; Fuzzy transform; F-transform

1. Introduction

Let $\mathbb{R}$ be the set of real numbers. A function $K : \mathbb{R} \to [0, 1]$ is said to be a *generating function*, if K is an even function that is non-increasing on $[0, \infty)$ and $K(x) > 0$ iff $x \in (-1, 1)$ holds true. A generating function K is said to be *normal* if $K(0) = 1$.

Triangular and raised cosine functions are typical examples of normal generating functions:

$$K_T(x) = \max(1 - |x|, 0)$$

and

$$K_C(x) = \begin{cases} \frac{1}{2}(1 + \cos(\pi x)), & -1 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Let K be a generating function, h and r be positive real numbers and $x_0 \in \mathbb{R}$. A system of fuzzy sets $\{A_i | i \in \mathbb{Z}\}$ on $\mathbb{R}$ defined by

$$A_i(x) = K\left(\frac{x - x_0 - ir}{h}\right) \quad \text{for} \quad i \in \mathbb{Z}$$

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is said to be a *generalized uniform fuzzy partition* (GUFP) of the real line determined by the quadruplet (K, h, r, x_0) if the *Ruspini condition* is satisfied:

$$S(x) = \sum_{i\in\mathbb{Z}} A_i(x) = 1 \tag{1}$$

holds for any $x \in \mathbb{R}$. The parameters h, r and x_0 are called the *bandwidth*, *shift* and the *central node*, respectively.

In [2], Holčapek et al. have proved a full characterization of generalized uniform fuzzy partitions by using the sum of suitable integrals.

Lemma 1. *(See [2].) A quadruplet (K, h, r, x_0) determines a generalized uniform fuzzy partition iff the quadruplet $(K, h, r, 0)$ determines it as well.*

Corollary 1. *(See [2].) Let $\beta > 0$ be a real number. A triplet (K, h, r) determines a generalized uniform fuzzy partition iff $(K, \beta h, \beta r)$ determines it as well.*

Let K be a normal generating function. Define $K_\alpha(x) = \alpha K(x)$ for $\alpha \in (0, 1]$, where $\alpha K(x)$ is the common product of real numbers. The necessary and sufficient condition for GUFPs can be significantly simplified in the cases of triangular and raised cosine generating functions have proved in [2].

Theorem 1. *(See [2].) Let $K \in \{K_T, K_C\}$. Then, $\left(K_{\frac{r}{h}}, h, r, x_0\right)$ determines a GUFP iff $\frac{h}{r} \in \mathbb{N}$.*

2. Counterexamples

Below we present Problem 7.1 from [3], which was posed by Holčapek et al. during the conference FSTA 2014.

Problem 1. (See [2].) Let K be a normal generating function satisfying the symmetry condition.

$$\int_{\frac{1}{2}-y}^{\frac{1}{2}+y} K(x)dx = y \quad \text{for all} \quad y \in \left[0, \frac{1}{2}\right] \tag{2}$$

Then $\left(K_{\frac{r}{h}}, h, r, x_0\right)$ determines a GUFP iff $\frac{h}{r} \in \mathbb{N}$.

The following counterexamples show that the necessary and sufficient condition for GUFP do not hold under the symmetric condition given by (2).

Counterexample 1 *(The counterexample of necessary condition).* Let K be a normal generating function defined by

$$K(x) = \begin{cases} 1, & x = 0 \\ \frac{3}{4}, & 0 < |x| \le \frac{1}{2} \\ \frac{1}{4}, & \frac{1}{2} < |x| < 1 \\ 0, & \text{otherwise} \end{cases}$$

And let $h = 1$, $r = \frac{1}{2}$ and $x_0 = 0$. Then K satisfies the symmetry condition and $\frac{h}{r} = 2 \in \mathbb{N}$. But

$$\begin{aligned} S(0) &= \sum_{i\in\mathbb{Z}} A_i(x) \\ &= \sum_{i\in\mathbb{Z}} K_{\frac{1}{2}}\left(\frac{0 - x_0 - ir}{h}\right) \\ &= \sum_{i\in\mathbb{Z}} \frac{1}{2} K\left(\frac{i}{2}\right) \end{aligned}$$