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A first-order epistemic quantum computational semantics with relativistic-like epistemic effects

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Abstract

Quantum computation has suggested new forms of quantum logic, called *quantum computational logics*. In these logics well-formed formulas are supposed to denote pieces of quantum information: possible pure states of quantum systems that can store the information in question. At the same time, the logical connectives are interpreted as quantum logical gates: unitary operators that process quantum information in a reversible way, giving rise to quantum circuits. Quantum computational logics have been mainly studied as *sentential logics* (whose alphabet consists of atomic sentences and of logical connectives). In this article we propose a semantic characterization for a *first-order epistemic quantum computational logic*, whose language can express sentences like "Alice knows that everybody knows that she is pretty". One can prove that (unlike the case of logical connectives) both quantifiers and epistemic operators cannot be generally represented as (reversible) quantum logical gates. The "act of knowing" and the use of universal (or existential) assertions seem to involve some irreversible "theoretic jumps", which are similar to quantum measurements. Since all epistemic agents are characterized by specific *epistemic domains* (which contain all pieces of information accessible to them), the unrealistic phenomenon of *logical omniscience* is here avoided: knowing a given sentence does not imply knowing all its logical consequences.

Keywords: Quantum computation; Quantum computational logics; Epistemic operators

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1. Introduction

The theory of quantum computation has inspired the development of new forms of quantum logics that have been termed *quantum computational logics*. As is well known, the basic idea of the theory of quantum computers is using as a "positive resource" two characteristic concepts of quantum theory that had been for a long time described as "mysterious" and potentially paradoxical: *superposition* and *entanglement*. In quantum computation any *piece*

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of information is identified with a possible *state* of a quantum system (say, a photon-system) that can store and transmit the information in question. In the happiest situations a state corresponds to a *maximal* piece of information (about the system) that cannot be consistently extended to a richer knowledge. Such states are called *pure*. Due to the characteristic indeterminism of quantum theory, a pure state is at the same time a *maximal* and a *logically incomplete* piece of information that cannot *decide* some important properties of the corresponding physical system. Accordingly, from an intuitive point of view, one can say that any pure state describes a kind of *cloud of potential properties* that might become *actual* when a measurement is performed, giving rise to the so-called *collapse of the wave-function*. The concept of *superposition* represents a mathematical realization of this intuitive idea. Any possible pure state of a quantum system S is identified with a unit-vector of an appropriate Hilbert space $\mathcal{H}_S$ and can be represented as a superposition of other unit-vectors that belong to a basis of the space. By adopting a notation introduced by Dirac, it is customary to write:

$$|\psi\rangle = \sum_i c_i |\varphi_i\rangle,$$

where c_i are complex numbers such that $\sum_i |c_i|^2 = 1$. The physical interpretation is the following: the system S that is in state $|\psi\rangle$ might satisfy the physical properties that are *certain* for the state $|\varphi_i\rangle$ with probability-value $|c_i|^2$. Apparently, any pure state $|\psi\rangle$ describes a *parallel* system of different pieces of quantum information ($|\varphi_i\rangle$). Just this parallelism is responsible for the extraordinary efficiency and speed of quantum computers.

Another powerful resource of quantum computation is due to the use of some "strange" pure states, called *entangled*, that turn out to violate the classical principle of *compositionality*. A paradigmatic case of entanglement may concern a composite physical system S consisting of two subsystems S_1 and S_2 (say, a two-electron system). The observer has a *maximal information* about S, represented by a pure state $|\psi\rangle$. What can be said about the states of the two subsystems? Due to the form of $|\psi\rangle$ and to the quantum-theoretic rules that concern the mathematical description of composite physical systems, such states cannot be pure: they are represented by two identical *mixed states*, which codify a "maximal degree" of uncertainty. Consequently, the information about the global systems (S) cannot be reconstructed as a function of the pieces of information about its parts (S_1, S_2). In such cases, information seems to flow from the *whole* to the *parts* (and not the other way around). Phenomena of this kind give rise to the so-called *holistic* features of quantum theory. Interestingly enough, entangled states are currently used in teleportation-experiments and in quantum cryptography.

As expected, quantum computation cannot be identified with a "static" representation of pieces of information. What is important is the dynamic *process* of information that gives rise to quantum computations (performed by *quantum circuits*). Such process is mathematically realized by *quantum logical gates* (briefly, *gates*): special examples of unitary operators that transform pure states into pure states in a reversible way. Since in quantum theory the time-evolution of physical systems is mathematically described by unitary operators, one can say that quantum computations can be regarded as the time-evolution of some special quantum objects.

Quantum computational logics can be described as a logical abstraction from the theory of quantum circuits. The basic idea that underlies the semantic characterization of these logics can be sketched as follows:

- well formed formulas are supposed to denote pieces of quantum information: possible states of quantum systems that can store the information in question;
- the logical connectives correspond to some gates that can process quantum information.

In this way, connectives turn out to have way a dynamic character, representing possible computation-actions. At the same time, any formula can be regarded as a synthetic logical description of a quantum circuit, which may have a characteristic parallel structure.

Quantum computational logics have been mainly studied as *sentential logics* (whose alphabet consists of atomic sentences and of logical connectives). Different choices of the system of primitive connectives and of the basic semantic definitions give rise to different logics. We will refer here to a *holistic* version of the quantum computational semantics, where quantum entanglement is used as a "semantic resource": generally, the *meaning* of a compound expression determines the *contextual meanings* of its subexpressions (and not the other way around, as happens in the case of most *compositional* semantic approaches).