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Fuzzy Sets and Systems 264 (2015) 121–137

FUZZY sets and systems

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Using discrete fuzzy numbers in the aggregation of incomplete qualitative information

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Received 16 December 2013; received in revised form 28 March 2014; accepted 12 May 2014

Available online 17 May 2014

Abstract

In this article discrete fuzzy numbers are used to model complete and incomplete qualitative information and some methods to aggregate this kind of information are proposed. When the support of discrete fuzzy numbers is a closed interval of the chain $L_n = \{0, 1, \ldots, n\}$, they can be interpreted as linguistic expert valuations that increase the flexibility of the elicitation of qualitative information based on linguistic terms. On the other hand, when the support is not an interval of L_n, the corresponding discrete fuzzy number can be interpreted as an incomplete linguistic expert valuation. In order to aggregate this kind of information, we propose two different methods. One of them deals with the construction of aggregation functions on the set of discrete fuzzy numbers from discrete aggregation functions defined on L_n. The other one presents several procedures for estimating the missing information based on the so-called *discrete associations*. Finally, the proposed aggregation methods are used in a multi-expert decision making problem and a concrete example is given.

Keywords: Aggregation function; Linguistic variable; Discrete fuzzy number; Subjective evaluation; Decision making

1. Introduction

The process of merging some data into a representative output is usually carried out by the so-called aggregation functions that have been extensively investigated in last decades [1,2,9,22]. Decision making, subjective evaluations, optimization and control are, among others, examples of concrete application fields where aggregation functions become an essential tool. In all these fields, it is well known that the data to be aggregated vary among many different kinds of information, from quantitative to qualitative information. Moreover, many times some uncertainty is inherent to such information.

Qualitative information is often interpreted to take values in a totally ordered finite scale like this:

$$\mathcal{L} = \{\text{Extremely Bad, Very Bad, Bad, Fair, Good, Very Good, Extremely Good}\}. \quad (1)$$

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http://dx.doi.org/10.1016/j.fss.2014.05.005

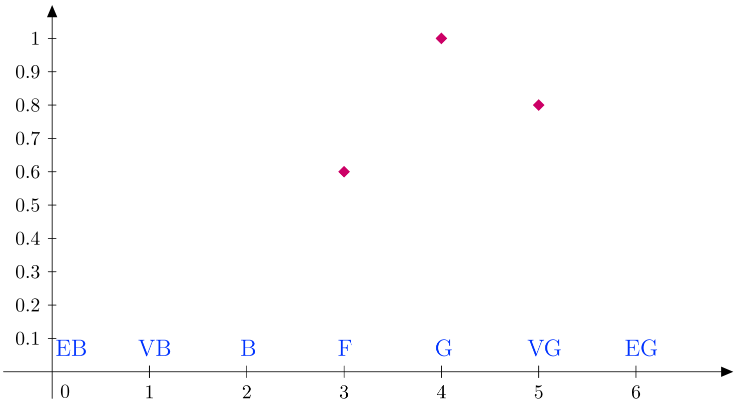

Fig. 1. Graphical representation of a discrete fuzzy number whose support is the interval [3, 5]. In addition, note that this fuzzy set can be interpreted as the expression "between Fair and Very Good", after identifying the linguistic scale $\mathcal{L}$ with the chain L_6.

In these cases, the representative finite chain $L_n = \{0, 1, \ldots, n\}$ is usually considered to model these linguistic hedges and several researchers have developed an extensive study of aggregation functions on L_n, usually called *discrete aggregation functions* (see [11,14,17]). Another approximation is based on assigning a fuzzy set to each linguistic term trying to capture its meaning. However, the modelling of linguistic information is limited because the information provided by experts for each variable must be expressed by a simple linguistic term, like *Bad, Good* or any other term in any linguistic scale similar to the one in (1). In most cases, this is a problem for experts because their opinion does not agree with one of these concrete terms. On the contrary, experts' values are usually expressions like "*better than Good*", "*between Fair and Very Good*" or even more complex expressions.[1]

To avoid the limitation above, some approaches have recently appeared trying to increase the flexibility of the elicitation of linguistic information (see [13,16,19–21]). One of these approaches [19] deals with the possibility of extending monotonic operations on L_n to operations on the set of discrete fuzzy numbers whose support is a set of consecutive natural numbers contained in L_n (i.e., an interval contained in L_n), usually denoted by $\mathcal{A}_1^{L_n}$, see [4,5,18,19]. The idea lies in the fact that any discrete fuzzy number $A \in \mathcal{A}_1^{L_n}$ can be considered (identifying the scale $\mathcal{L}$ given in (1) with L_n with $n = 6$) as an assignation of a $[0, 1]$-value to each term in our linguistic scale. As an example, the above mentioned expression "*between Fair and Very Good*" can be performed, for instance, by a discrete fuzzy number $A \in \mathcal{A}_1^{L_6}$, with support given by the subinterval $[F, VG]$ (that corresponds to the interval $[3, 5]$ in L_6). The values of A in its support should be described by experts, allowing in this way a complete flexibility of the qualitative valuation. A possible discrete fuzzy number A representing the expression mentioned above is given in Fig. 1 (note that only the values of A in its support are pictured).

Thus, aggregation functions on $\mathcal{A}_1^{L_n}$ will allow us to manage qualitative information in a more flexible way. In [4] t-norms and t-conorms on $\mathcal{A}_1^{L_n}$ are described and studied, as well as it is done for uninorms, nullnorms and general aggregation functions in [18]. In both cases, an example of application in decision making or subjective evaluation is included.

In this way, discrete fuzzy numbers in $\mathcal{A}_1^{L_n}$ can be interpreted as flexible qualitative information and they have been successfully used in decision making problems and subjective evaluation. On the other hand, note that when we take a discrete fuzzy number A whose support is not an interval of L_n (that is, being $A \notin \mathcal{A}_1^{L_n}$), say for instance $supp(A) = \{i_1, i_2, \ldots, i_k\}$, there are some items in the interval $[i_1, i_k]$ that have no assigned value. These gaps in the support can be interpreted as lacks of information and any discrete fuzzy number of this type as an incomplete qualitative information. More specifically, in any expert's opinion expressed as a discrete fuzzy number, like *"between fair and very good"*, it is expected that the support of the corresponding discrete fuzzy number includes at least the terms "Fair, Good, Very Good". If, for instance the terms "Fair" and "Very Good" are in the support, but the term

[1] To deal with these expressions one can use fuzzy numbers in general, but then a process of fuzzyfication and defuzzyfication is needed, which can be completely avoided using the approach based on discrete fuzzy numbers proposed in this paper (see also [18,19]).