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Fuzzy Sets and Systems 294 (2016) 44–47

FUZZY sets and systems

www.elsevier.com/locate/fss

Short communication

A note on a fuzzy rough set model for set-valued data

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Received 20 September 2014; received in revised form 11 March 2015; accepted 12 March 2015

Available online 18 March 2015

Abstract

Dai and Tian recently introduced the notion of fuzzy rough set model for set-valued information system. In this paper, it is pointed out that the family of reducts defined by Dai and Tian need not be a subset of the family of reducts defined within the standard rough set model for set-valued information system. In fact, there is no relation between the numbers of reducts obtained in both approaches.

Keywords: Set-valued information systems; Fuzzy rough set model for set-valued information system; Attribute reduction

1. Introduction

Set-valued information systems [8,10] whose attribute values are set-valued, are further generalizations of single-valued information systems and applied to characterize uncertainty and missing information in [1]. Moreover, incomplete information systems have been handled by set-valued information systems in [3,7], where each unknown value for a certain attribute is replaced with the set of all possible values for that attribute. According to [1,2,6], there are two different views of sets following from the semantic meaning: the disjunctive and conjunctive views. In the disjunctive view, a set is represented as a disjunction of possible items or values of some underlying quantity, which means that one of them is the right one. In the conjunctive view, a set is a collection of items, which has an objective existence as a lumped entity. Therefore, the set-valued attribute follows from the conjunctive view in a general way. Once incomplete information systems are dealt in the framework of set-valued information systems, the set replacing an unknown attribute is a disjunction of all possible values. Dubois and Prade [4,5] first proposed fuzzy rough set model to combine fuzzy sets [11] and rough sets [9], which has not been applied into set-valued information systems. Hence, Dai and Tian [3] introduced a specific technique to define a fuzzy tolerance relation between set values. Dai and Tian named that method fuzzy rough set model for set-valued information system to distinguish it from the existing fuzzy rough set model. In the sequel, we call the Dai and Tian approach fuzzy rough set approach, and the other one rough set approach.

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http://dx.doi.org/10.1016/j.fss.2015.03.007

In this paper, we show that the family of reducts defined within fuzzy rough set approach is not necessarily a subset of the family of reducts defined within rough set approach.

2. Fuzzy rough set model for set-valued information system

Let a quadruple $IS = \langle U, A, V, f\rangle$ be an information system. Then IS is called a *single-valued information system*, when $f(a, x)$ is a single value for all $a \in A, x \in U$. Otherwise, IS is called a *set-valued information system*. For brevity, $f(a, x)$ is also written as $a(x)$, and denotes a set in a set-valued information system. For each crisp set X, $|X|$ denotes the cardinality of the set X.

Definition 2.1. (See Dai and Tian [3].) Let $\langle U, A, V, f\rangle$ be a set-valued information system. Then for all $a \in A$, a fuzzy tolerance relation $\widetilde{R_a}$ is defined as

$$\widetilde{R_a}(x, y) = \frac{|a(x) \cap a(y)|}{|a(x) \cup a(y)|} \text{ for all } x, y \in U.$$

Let P be a non-empty subset of A. Then a fuzzy tolerance relation $\widetilde{R_P}$ is defined as

$$\widetilde{R_P}(x, y) = \inf_{a\in P} \widetilde{R_a}(x, y) \text{ for all } x, y \in U.$$

Moreover, P is a *reduct* of A in fuzzy rough set approach if $\widetilde{R_A}(x, y) = \widetilde{R_P}(x, y)$ for all $x, y \in U$ and for any P' satisfying $\emptyset \neq P' \subset P$, there exist $x, y \in U$ such that $\widetilde{R_A}(x, y) \neq \widetilde{R_{P'}}(x, y)$. The family of all reducts of A in fuzzy rough set approach is denoted by $Red(A)$. The discernibility matrix M is defined as $M_{n\times n} = (M_{ij})_{n\times n}$, where $n = |U|$ and M_{ij} is a set of attributes for all $a \in A$, $a \in M_{ij}$ iff $\widetilde{R_a}(u_i, u_j) = \widetilde{R_A}(u_i, u_j)(u_i, u_j \in U)$.

Let the fuzzy relations $\widetilde{R_a}$ and $\widetilde{R_P}$ reduce to crisp relations T_a and T_P, respectively, as

$$T_a(x, y) = \begin{cases} 1, & a(x) \cap a(y) \neq \emptyset; \\ 0, & a(x) \cap a(y) = \emptyset; \end{cases} \text{ and } T_P(x, y) = \inf_{a\in P} T_a(x, y) \text{ for all } x, y \in U.$$

Then we obtain rough set model for set-valued information system, i.e., rough set approach. The family of all reducts of A in rough set approach is denoted by $Red'(A)$. The discernibility matrix M' in rough set approach is defined as $M'_{n\times n} = (M'_{ij})_{n\times n}$, where for all $a \in A$, $a \in M'_{ij}$ iff $T_a(u_i, u_j) = T_A(u_i, u_j)(u_i, u_j \in U)$.

Theorem 2.2. *(See Dai and Tian [3].) Let $\langle U, A, V, f\rangle$ be a set-valued information system and $P \subseteq A$. Then P is a reduct of A in fuzzy rough set approach if and only if P is a minimal set satisfying $P \cap M_{ij} \neq \emptyset$ for all $u_i, u_j \in U$.*

Theorem 2.3. *Let $\langle U, A, V, f\rangle$ be a set-valued information system and $P \subseteq A$. Then P is a reduct of A in rough set approach if and only if P is a minimal set satisfying $P \cap M'_{ij} \neq \emptyset$ for all $u_i, u_j \in U$.*

The proof of Theorem 2.3 is similar to that of Theorem 3.1 in [3].

Lemma 2.4. *(See Dai and Tian [3].) Let $\langle U, A, V, f\rangle$ be a set-valued information system. Then $M_{ij} \subseteq M'_{ij}$ for all $u_i, u_j \in U$.*

Definition 2.5. Let $\mathcal{S}$ be a nonempty subset of power set of A. Then a nonempty set P is called a *minimal element* of $\mathcal{S}$, if $P \in \mathcal{S}$ and $P' \nsubseteq P$ for any other $P' \in \mathcal{S}$. The family of all minimal element(s) of $\mathcal{S}$ is denoted by $Min(\mathcal{S})$.

Theorem 2.6. *Let $\langle U, A, V, f\rangle$ be a set-valued information system,*

$$\mathcal{S} = \{P \subseteq A | P \cap M_{ij} \neq \emptyset \text{ for all } u_i, u_j \in U\} \text{ and } \mathcal{S}' = \{P \subseteq A | P \cap M'_{ij} \neq \emptyset \text{ for all } u_i, u_j \in U\}.$$

Then $Red(A) = Min(\mathcal{S})$ and $Red'(A) = Min(\mathcal{S}')$. Moreover, for each $P \in Red(A)$, there exists $Q \in Red'(A)$ such that $Q \subseteq P$.

Proof. It follows immediately from Theorems 2.2, 2.3 and Definition 2.5 that $Red(A) = Min(\mathcal{S})$ and $Red'(A) = Min(\mathcal{S}')$. We have $\mathcal{S} \subseteq \mathcal{S}'$ by Lemma 2.4. Since the power set of A is a lattice and $P \in Min(\mathcal{S})$, there exists $Q \in Min(\mathcal{S}') = Red'(A)$ such that $Q \subseteq P$. $\square$