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Short Communication

Attribute-oriented fuzzy concept lattices and standard fuzzy concept lattices induce the same similarity on objects ☆

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Abstract

G. Ciobanu, C. Văideanu (2015) [1] define the similarity of objects in attribute-oriented fuzzy concept lattice and claim that it is different from the similarity of objects induced by the corresponding fuzzy context. We show that this claim is wrong and that the two similarities are equal. As a consequence we get that attribute-oriented concept lattices and standard concept lattices induce the same similarity on objects.

Keywords: Formal concept analysis; Fuzzy logic; Attribute-oriented concept lattice; Similarity

Ciobanu and Văideanu [1] define similarity of objects induced by attribute-oriented fuzzy concept lattice and show that it is a superrelation of similarity induced by the corresponding fuzzy context. They incorrectly claim that the opposite inclusion does not hold true, in general. The goal of this paper is to correct this misconception. We show that the opposite inclusion generally holds true as well, and thus the induced similarities are the same as in the case of standard concept lattices.

We use the same notations as [1]; for preliminaries see [1].

Similarity of objects in standard concept lattice $\mathcal{B}(X, Y, R)$ and similarity of objects in attribute-oriented concept lattice $\mathcal{B}_p(X, Y, R)$ is defined as

$$E^X_{\mathcal{B}(X,Y,R)}(x_1, x_2) = \bigwedge_{(A,B)\in\mathcal{B}(X,Y,R)} (A(x_1) \leftrightarrow A(x_2)), \tag{1}$$

$$E^X_{\mathcal{B}_p(X,Y,R)}(x_1, x_2) = \bigwedge_{(A,B)\in\mathcal{B}_p(X,Y,R)} (A(x_1) \leftrightarrow A(x_2)), \tag{2}$$

respectively.

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The similarity (1) is the induced (by $\mathcal{B}(X, Y, I)$) similarity on X defined by Belohlavek in [2, (5.6)], while the similarity (2) is the similarity defined in [1, Definition 6].

Belohlavek in [2] proves that $E^X_{\mathcal{B}(X,Y,R)}$ is equivalent to fuzzy equivalence[1] induced by the fuzzy context (X, Y, R), which is defined as

$$E^X_{(X,Y,R)}(x_1, x_2) = \bigwedge_{y \in Y} (R(x_1, y) \leftrightarrow R(x_2, y)) . \tag{3}$$

Theorem 1. *(See Theorem 5.12 in [2].) Let (X, Y, R) be a fuzzy context. Then for the similarities (1) and (3) we have*

$$E^X_{\mathcal{B}(X,Y,R)} = E^X_{(X,Y,R)}.$$

Ciobanu and Văideanu [1, Theorem 9] show that for the similarity relations defined by (2) and (3) we have

$$E^X_{\mathcal{B}_p(X,Y,R)} \supseteq E^X_{(X,Y,R)} \tag{4}$$

and that double negation is sufficient to make (4) an equality analogous to the one in Theorem 1. We have now proven that the equality actually holds true in general.

Theorem 2. *Let (X, Y, R) be a fuzzy context. Then for the similarity relations defined by (2) and (3) we have*

$$E^X_{\mathcal{B}_p(X,Y,R)} = E^X_{(X,Y,R)}.$$

Proof. By definition of $E^X_{\mathcal{B}_p(X,Y,R)}$ and the fact that extents of $\mathcal{B}_p(X, Y, R)$ are exactly fuzzy sets of the form $B^\square$, $B \in L^Y$, we have

$$\begin{aligned} E^X_{\mathcal{B}_p(X,Y,R)}(x_1, x_2) &= \bigwedge_{(A,B) \in \mathcal{B}_p(X,Y,R)} (A(x_1) \leftrightarrow A(x_2)) \\ &= \bigwedge_{B \in L^Y} \left(B^\square(x_1) \leftrightarrow B^\square(x_2)\right) . \end{aligned}$$

Note that each $B \in L^Y$ can be written as the intersection

$$B = \bigcap_{y \in Y} B_{y,B(y)}$$

where the fuzzy sets $B_{y,k}$ ($y \in Y, k \in L$) are defined as

$$B_{y,k}(y') = \begin{cases} k & \text{if } y = y', \\ 1 & \text{otherwise.} \end{cases}$$

We have

$$\begin{aligned} E^X_{\mathcal{B}_p(X,Y,R)}(x_1, x_2) &= \bigwedge_{B \in L^Y} \left((\bigcap_{y \in Y} B_{y,B(y)})^\square(x_1) \leftrightarrow (\bigcap_{y \in Y} B_{y,B(y)})^\square(x_2) \right) \\ &= \bigwedge_{B \in L^Y} \left((\bigcap_{y \in Y} B^\square_{y,B(y)})(x_1) \leftrightarrow (\bigcap_{y \in Y} B^\square_{y,B(y)})(x_2) \right) \\ &= \bigwedge_{B \in L^Y} \left(\bigwedge_{y \in Y} B^\square_{y,B(y)}(x_1) \leftrightarrow \bigwedge_{y \in Y} B^\square_{y,B(y)}(x_2) \right) \end{aligned}$$

[1] Called attribute-oriented similarity between objects in [1].