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Relations of reduction between covering generalized rough sets and concept lattices

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ABSTRACT

The reduction theory plays an important role in data analysis. This paper studies the relation between the reduction of a covering and the attribute reduction of a concept lattice. The reduction of a covering from the perspective of concept lattices is investigated. Conversely, the attribute reduction of a formal context is studied in the framework of covering generalized rough sets. The results in this paper show that the reduction of a covering can be viewed as the attribute reduction of a derivative formal context. Moreover, every reduct of a given formal context can be seen as the reduct of an induced covering. As an application of the theoretical results, an approach to the attribute reduction of concept lattices based on covering generalized rough sets is proposed. Furthermore, experiments are given to show the effectiveness of the proposed method.

1. Introduction

Various approaches have been proposed to discover non-trivial, novel, previously unknown and potentially useful knowledge in data mining. For example, fuzzy set theory [59], rough set theory [30,31], formal concept analysis (also called concept lattice theory) [4,28,49] and many others have been developed and applied in diverse fields [6,15,17,33,35–37,39,51]. Of all paradigms, rough set theory (RST) and formal concept analysis (FCA) are two related and often complementary methods for data analysis.

RST is an extension of the classical set theory for the study of uncertain and incomplete information. The basic notion of RST is an equivalence relation on a set of objects called the universe. However, such an equivalence relation is still restrictive for many applications [58]. To address this issue, the classical rough set model is extended to relation based rough sets [29,42,43] and covering generalized rough sets [2,62,63]. Two fundamental notions in FCA are formal context and formal concept. The classical concept lattice has been generalized to object and attribute oriented concept lattices [23,26,55]. In recent years, more attention has been paid to comparing and combining the two theories [9,18,47,57]. For example, one can introduce the rough set approximation operators into FCA [7,16]. Conversely, one can also introduce the notion of concept lattice into RST [5,8]. These results have improved our understanding of their similarities and differences [56].

The notion of reduction is a basic issue in knowledge discovery [19,20]. In the classical rough sets, the reduction is to find a minimal attribute set which can keep the partition of the universe. While in covering generalized rough sets, there are two types of covering reduction [54]. One is to reduce redundant elements of a covering and find a minimal subset of coverings

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that induces the same covering lower and upper approximations [62]. The other is to reduce redundant coverings from a family of coverings [3,11,46,61], which is related to the attribute reduction of rough sets. Generally speaking, these two covering reduction are different concepts with different purposes. In this paper, we pay our attention on the first type of covering reduction theory. In FCA, there are many interesting results in the attribute reduction [13,14,21,24,25,40,41,45,61]. In [4], a reduction of a context by deleting rows or columns was introduced. The main idea of this approach was to use a classification of the attributes based on the irreducible element. In 2005, Zhang et al. [60] proposed a new notion of attribute reduction which can preserve all of the concepts and their hierarchy in a context. In addition, they presented a method for the attribute reduction based on the discernibility matrix and function. After that, Wei [48] proposed an effective method for computing the attribute reduction of a concept lattice. Based on the granular computing theory [1,32], Wu et al. used the information granules to explore the attribute reduction in FCA [50]. Recently, Medina [27] studied the attribute reduction among object-oriented and property-oriented concept lattices, and formal concept lattices. According to the result in [27], one only needs to consider the attribute reduction in one of the three concept lattices. Furthermore, an algorithm was designed to obtain the attribute reduction of concept lattices. In [47], the relation of reduction between the classical concept lattice and the Pawlak's rough set was researched. However, little attention has been paid to the reduction of covering generalized rough sets from a concept lattice point of view.

Following with the notion of $\wedge$-irreducible element presented by Ganter and Wille [4], this paper studies the relation of reduction between covering generalized rough sets and the classical concept lattices, which can make us understand the relation between the two different reduction mechanisms. The remainder of this paper is organized as follows. In Section 2, some basic notions regarding generalized rough sets and concept lattices are reviewed. In Section 3, a new concept of reduction called intersection reduction of a covering is introduced, and its properties are also examined. In Section 4, we investigate the reduction of a covering from the viewpoint of concept lattice theory. A new method for computing all the reducts of a concept lattice based on covering generalized rough sets is presented in Section 5. In Section 6, an algorithm is constructed to compute the reduction of concept lattices. In addition, numerical experiments are given to show the effectiveness of the proposed technique. Finally, some conclusions are drawn in Section 7.

2. Preliminaries

In this section, we review some basic notions of covering generalized rough sets and concept lattices [2,62,63].

2.1. Covering generalized rough sets

Let U, the universe of discourse, be a non-empty finite set. We use $P(U)$ to denote the power set of U and X^c to denote the complement of X in U.

Definition 2.1 ([2]). Let $\mathcal{C}$ be a family of subsets of the universe U. $\mathcal{C}$ is called a covering of U if none elements in $\mathcal{C}$ is empty and $\cup_{K\in\mathcal{C}}K = U$. The ordered pair $(U, \mathcal{C})$ is said to be a covering approximation space (CA-space).

It is clear that a partition of U is certainly a covering of U, so the concept of a covering is an extension of the concept of a partition.

Definition 2.2 ([63]). Let $(U, \mathcal{C})$ be a CA-space. For $x \in U$, the neighborhood of x with respect to $\mathcal{C}$ is defined as:

$$N_{\mathcal{C}}(x) = \cap\{K \in \mathcal{C} : x \in K\}.$$

The family of all neighborhoods with respect to $\mathcal{C}$ is defined as:

$$N_{\mathcal{C}} = \{N_{\mathcal{C}}(x) : x \in U\}.$$

Most researches on covering generalized rough sets mainly focus on the studies of covering approximation operators. There are many kinds of covering approximation operators in the literature [2,22,34,53,62,63]. For example, Qin et al. [38] and Zhu [63] proposed five pairs of approximations by means of the neighborhood. For our purpose, we only recall the following covering approximation operators.

Definition 2.3 ([53]). Let $(U, \mathcal{C})$ be a CA-space. The operations $L_{\mathcal{C}} : P(U) \to P(U)$ and $H_{\mathcal{C}} : P(U) \to P(U)$ are defined as: for $X \subseteq U$,

$$L_{\mathcal{C}}(X) = \{x \in U : N_{\mathcal{C}}(x) \subseteq X\}, H_{\mathcal{C}}(X) = \{x \in U : N_{\mathcal{C}}(x) \cap X \neq \emptyset\}.$$

$L_{\mathcal{C}}$ and $H_{\mathcal{C}}$ are called the lower approximation and the upper approximation with respect to $\mathcal{C}$, respectively.

The notion of reduction is one of the main problems of covering generalized rough sets. In what follows, we review the reduction theory proposed by Zhu and Wang [62], which is also called the union reduction theory in [54].

Definition 2.4 ([62], *Union reducible covering*). Let $(U, \mathcal{C})$ be a CA-space and $K \in \mathcal{C}$. If K is a union of some sets in $\mathcal{C} - \{K\}$, we say that K is a reducible (or union reducible) element of $\mathcal{C}$; otherwise, we say that K is an irreducible (or a union irreducible) element of $\mathcal{C}$.