Information Sciences 312 (2015) 25–39

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

An Information Axiom based decision making approach under hybrid uncertain environments

Dongping Chen [a], Xuening Chu [a,*], Xiwu Sun [b], Yupeng Li [c], Yuliang Su [a]

[a] School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai, China
[b] Shanghai Aerospace Equipments Manufacturer, Shanghai, China
[c] China University of Mining and Technology, Xuzhou, Jiangsu, China

ARTICLE INFO

Article history:
Received 7 November 2013
Received in revised form 16 March 2015
Accepted 20 March 2015
Available online 28 March 2015

Keywords:
Multiple criteria analysis
Information Axiom
Hybrid uncertainty
Information Content model
Expected value model
Chance constrains programming model

ABSTRACT

This paper aims to develop a decision making approach based on the Information Axiom under hybrid uncertain environments. In these environments fuzziness and randomness co-exist. To model the hybrid uncertainties, the design range and system range of an evaluation criterion are expressed as a fuzzy variable and a random variable respectively. Calculating the Information Contents for these criteria would involve integrals whose bounds are fuzzy variables. To calculate these integrals, an expected Information Content Model (eICM) is proposed based upon the credibility theory. To reflect the designer's risk appetite accurately, the confidence levels are introduced and a credibility Information Content Model (cICM) based on the chance constrains programming model is constructed. Two algorithms based on fuzzy simulation methods and Genetic Algorithm are developed for these two models. Finally, a case study is illustrated to highlight the effectiveness and accuracy of the proposed approach.

1. Introduction

Axiomatic design developed by Suh [37] was motivated by the absence of scientific design principles [7]. According to axiomatic design, a design work could be regarded as a zigzag mapping process between the Functional Requirements (FRs) and the Design Parameters (DPs). Two fundamental design principles, the Independence Axiom and the Information Axiom should be satisfied in this mapping. The Independence Axiom maintains the independence of FRs and enhances the controllability of a design. The Information Axiom is used to minimise the Information Content of a design and reduce the design complexity. The Information Content is defined as a measure of complexity, and it is related to the probability of conceived solutions meeting FRs. The objective of this paper is to extend the Information Axiom under hybrid uncertain environments characterized by randomness and fuzziness co-existing. Randomness is about an occurrence (or non-occurrence) of a certain event (typically described by some set) [29]. Fuzziness is not about an occurrence but about a degree of membership to a certain concept [29].

Randomness and fuzziness sometimes co-exist in actual decision making problems. Take the evaluation criterion "timeliness" in a service selecting problem for example. On the one hand, "timeliness" is a random variable obeying some pdf (probability distribution function) for the reason that each execution of this service happens at different time and different

* Corresponding author.
E-mail address: xnchu@sjtu.edu.cn (X. Chu).

http://dx.doi.org/10.1016/j.ins.2015.03.054
0020-0255/

places by different staffs, and the pdf is denoted as $f(FR)$. On the other hand, "timeliness" is usually described in imprecise linguistic terms such as "very slow", "slow", "medium", "fast", "very fast", which could be modelled by the fuzzy set theory [28,39,42,43]. We refer to this kind of criterion as a hybrid uncertain criterion [24]. Two difficult issues will be encountered under hybrid uncertain environments. The first one is how to calculate the integrals whose bounds are fuzzy variables. The integrals are used to calculate the probability of a design solutions meeting FRs (successful probability) which is related to the Information Content. For a hybrid uncertain criterion, the integral bounds are fuzzy variables. For example, the success probability of "timeliness" is $\int_{\text{"medium"}}^{\text{"very fast"}} f(FR)dFR$, and the integral bounds are two fuzzy variables "medium" and "very fast". How to calculate this kind of integrals characterised by fuzzy bounds is seldom mentioned in existing researches. The second problem is how to reflect the risk appetite of decision makers in the Information Content. Due to the multi-value characteristic of fuzzy variables, the success probability of a hybrid uncertain criterion is also multi-value. Take the success probability of "timeliness" $\int_{\text{"medium"}}^{\text{"very fast"}} f(FR)dFR$ for example again. The up and down fuzzy bound "very fast" and "medium" are both multi-value. Every combination of the up and down bound's value determines a success probability of "timeliness". In order to calculate the Information Content, an integrate value should be obtained from the multiple values, which is influenced by decision makers' risk appetite. The bigger the integrated value takes, the more risky the decision makers' risk appetite is. Up to now, little research deals with these problems. The existing Information Axiom approaches, considering randomness or fuzziness alone, could not describe the reality perfectly, and the evaluation results could not express the opinions of decision makers accurately.

An extension of the Information Axiom under hybrid uncertain environments is presented in this paper. The remaining part is organized as follows. Section 2 reviews the extensive literatures about the Information Axiom and the credibility theory. In Section 3 some important concepts are represented. In Section 4 two Information Content models are developed. Section 5 demonstrates the application of the proposed evaluation approach by a case. Section 6 compares the proposed approach with the traditional Information Axiom and concludes this paper.

2. Literature review

According to the uncertainty types of the system and design range, the Information Axiom approaches can be classified into three categories: the traditional, fuzzy and hybrid uncertain Information Axiom (shown in Table 1). The traditional Information Axiom was proposed by Suh [37], handling the criteria with crisp design ranges and crisp/random system range. Decision making problems with quantitative criteria evaluation can be solved by the traditional Information Axiom [10,18,38]. The fuzzy Information Axiom was developed by Kulak and Kahraman [19,20] and both the system range and design range with incomplete information were considered as fuzzy variables. The common range was the intersection area of triangular or trapezoidal fuzzy numbers. The Information Content model (ICM) under a fuzzy environment was also developed. The fuzzy Information Axiom has been widely used to deal with qualitative criteria [2,9,15,16,44]. Gören and Kulak [9] extended the fuzzy axiomatic design approach to including risk factors through diminishing the common range. The above two kinds of Information Axiom approaches ignored the hybrid uncertainties in design evaluation problems, and merely considered randomness or fuzziness alone. The hybrid uncertain criteria with fuzzy design ranges and random system ranges are rarely discussed in the existing researches.

The integral whose bounds are fuzzy variables is essentially a kind of uncertain function, which could be solved by the possibility theory and credibility theory. The possibility theory is based on two non-additive measures: possibility and necessity measures, and it has been widely used to resolve fuzzy linear programming problems [3,8,13,14,23,27,36,40]. However, the possibility measure has no self-duality property [6]. According to respective definitions, a fuzzy event may fail even though its possibility is 1, and hold even though its necessity is 0 [25]. Therefore, Liu and Liu [26] proposed the credibility theory, and stated that the credibility measure played the role of probability measure in a fuzzy decision system. Based on the credibility theory, the expected value model (EVM) of a fuzzy variable [26] which is essentially a type of Choquet integral, has been widely used in fuzzy programming problems, such as portfolios selecting problems [12,24,40], project

Table 1
The classifications of Information Axiom and the research status.

	Traditional Information Axiom	Fuzzy Information Axiom	Hybrid uncertain Information Axiom
Design range	Crisp intervals (quantitative)	Fuzzy variables (qualitative)	Fuzzy variables (qualitative)
System range	Crisp intervals or random variables (random)	Fuzzy variables (qualitative)	Random variables (random)
Information Content models	$I = -\log_2 p = -\log_2\left(\frac{\text{Common Range}}{\text{System Range}}\right)$	$I = -\log_2 p = -\log_2\left(\frac{\text{fuzzy Common Range}}{\text{fuzzy System Range}}\right)$	$I = -\log_2 p = -\log_2 \int_{\tilde{A}_1}^{\tilde{A}_2} f(x)d(x)$ where, $\tilde{A}_1, \tilde{A}_2$ are fuzzy numbers (proposed in this paper)
Application scope	Quantitative criteria	Qualitative criteria	Hybrid uncertain criteria
Characteristics	Cannot deal with uncertain criteria	can deal with fuzziness only	Can deal with fuzziness and randomness simultaneously
Research status	[10,18,38]	[1,5,41]	None