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State feedback based output tracking control of probabilistic Boolean networks☆

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ABSTRACT

This paper studies the state feedback based output tracking control of probabilistic Boolean control networks (PBCNs) by using the semi-tensor product of matrices. A series of reachable sets (with probability 1) is defined inductively for PBCNs, and some useful properties are obtained for the reachable sets. Based on the properties of the reachable sets, a necessary and sufficient condition is presented for the state feedback based output tracking control of PBCNs. Meanwhile, a constructive procedure is proposed to design state feedback laws for PBCNs to track a constant reference signal. The study of two illustrative examples shows that the obtained new results are very effective.

1. Introduction

As a generalization of deterministic Boolean networks [1–3], probabilistic Boolean networks (PBNs) were introduced by Shmulevich et al. [4] to model gene regulatory networks in 2002. PBNs consist of a set of Boolean networks and a probability structure which regulates the switching between different constituent Boolean networks. The main advantage of the PBN model over the deterministic Boolean network is that it cannot only share the appealing properties of Boolean networks but also cope with the presence of uncertainties. In the last decade, the study of PBNs has attracted a great attention from biologists and systems scientists [5–11].

As was described in [12], "Gene regulatory networks are defined by trans and cis logic. ... Both of these types of regulatory networks have input and output." PBNs with input and output are called probabilistic Boolean control networks (PBCNs). The control of PBCNs is a very important issue because it can provide the (optimal) therapeutic ways for the treatment of some diseases. Thus, in the last decade, many scholars have put their efforts to find suitable control strategies for PBCNs. The optimal control problem of PBCNs was investigated in [13,14], and some effective methods were proposed for the optimal control design. The state feedback stabilization problem of PBCNs was studied in [15,16], and a constructive procedure was established for the design of state feedback stabilizers.

Recently, based on the semi-tensor product of matrices, an algebraic state space representation (ASSR) framework [17,18] has been established for the study of both deterministic Boolean networks and PBNs. Using the ASSR framework, one can

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convert the dynamics of a Boolean (control) network into a linear (bilinear) discrete-time system, and then one can investigate Boolean (control) networks by using the classical control theory. Until now, there have been lots of excellent results on the control of deterministic Boolean networks [19–26] and PBCNs [15,27–31] via the ASSR framework.

As is well known, one of the most fundamental issues in the control theory is the output tracking problem, which aims to design suitable control strategies such that the measured outputs of a plant track a desirable reference signal [32]. It should be pointed out that the output tracking control problem is also relevant for gene regulatory networks. For example, in order to manipulate the large scale behavior of the lactose regulation system of the Escherichia coli bacteria, Julius et al. [33] proposed a novel feedback control architecture to make the fraction of induced cells in the population (the output of the system) attain a desired level (a given reference signal). The output tracking control problem of deterministic Boolean networks was studied in [34,35], respectively, and some constructive procedures were presented for the design of output tracking controllers. However, to our best knowledge, there are no results available on the output tracking control of PBCNs. In addition, due to the stochastic nature of PBCNs, the methods of solving the output tracking control problem of deterministic Boolean networks cannot be directly applied to study the output tracking control problem of PBCNs.

In this paper, using the ASSR framework, we investigate how to design state feedback based output tracking controllers for PBCNs. We define inductively a series of reachable sets with probability 1 for PBCNs, and obtain some useful properties of the reachable sets. Based on these properties, we present a necessary and sufficient condition for the state feedback based output tracking control of PBCNs. In addition, we propose a computationally tractable procedure to design state feedback laws for PBCNs to track a constant reference signal. Finally, we apply the obtained new results to the bistability analysis of apoptosis networks.

The rest of this paper is organized as follows. Section 2 formulates the state feedback based output tracking control problem of PBCNs. Section 3 presents the main results of this paper. Two illustrative examples are given to support our new results in Section 4, which is followed by a brief conclusion in Section 5.

Notation: $\mathbb{R}$, $\mathbb{N}$ and $\mathbb{Z}_+$ denote the sets of real numbers, natural numbers and positive integers, respectively. $\mathcal{D} := \{1, 0\}$. $\Delta_n := \{\delta_n^k : k = 1, \dots, n\}$, where δ_n^k denotes the kth column of the identity matrix I_n. For compactness, $\Delta := \Delta_2$. An $n \times t$ matrix M is called a logical matrix, if $M = [\delta_n^{i_1}\ \delta_n^{i_2}\ \dots\ \delta_n^{i_t}]$, and we express M briefly as $M = \delta_n[i_1\ i_2\ \dots\ i_t]$, and denote the set of $n \times t$ logical matrices by $\mathcal{L}_{n\times t}$. $Col_i(A)$ denotes the ith column of the matrix A, and $Row_i(A)$ stands for the ith row of the matrix A. $Blk_i(A)$ denotes the ith $n \times n$ block of an $n \times mn$ matrix A. "$\neg$", "$\wedge$" and "$\vee$" denote Negation, Conjunction and Disjunction, respectively.

2. Problem formulation

Consider the following probabilistic Boolean control network:

$$\begin{cases} X(t+1) = f(X(t), U(t)), \\ Y(t) = h(X(t)), t \in \mathbb{N}, \end{cases} \tag{2.1}$$

where $X(t) = (x_1(t), x_2(t), \dots, x_n(t)) \in \mathcal{D}^n$, $U(t) = (u_1(t), \dots, u_m(t)) \in \mathcal{D}^m$ and $Y(t) = (y_1(t), \dots, y_p(t)) \in \mathcal{D}^p$ are the state, the control input and the output of the system (2.1), respectively. $f : \mathcal{D}^{m+n} \mapsto \mathcal{D}^n$ is chosen from the set $\{f_1, f_2, \dots, f_r\}$ at every time step, and $P\{f = f_i\} = p_i > 0$, where $f_i : \mathcal{D}^{m+n} \mapsto \mathcal{D}^n, i = 1, 2, \dots, r$ are given logical functions, and $\sum_{i=1}^r p_i = 1$. $h : \mathcal{D}^n \mapsto \mathcal{D}^p$ is a given logical function.

Given a constant reference signal $Y^* = (y_1^*, \dots, y_p^*) \in \mathcal{D}^p$. For the PBCN (2.1), *the state feedback based output tracking control problem* is to design a state feedback control in the form of

$$U(t) = g(X(t)), \tag{2.2}$$

under which there exists an integer $T > 0$ such that

$$P\{Y(t) = Y^* \mid X(0) = X_0, U(t) = g(X(t))\} = 1 \tag{2.3}$$

holds for $\forall\ X_0 \in \mathcal{D}^n$ and $\forall\, t \geq T$, where $g : \mathcal{D}^n \mapsto \mathcal{D}^m$ is a logical function to be designed.

In order to convert the system (2.1) and the state feedback control (2.2) into equivalent algebraic forms, respectively, we recall the definition and some useful properties of the semi-tensor product of matrices. For details, please refer to [17,18].

Definition 2.1 [17]**.** The semi-tensor product of two matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ is

$$A \ltimes B = (A \otimes I_{\frac{\alpha}{n}})(B \otimes I_{\frac{\alpha}{p}}), \tag{2.4}$$

where $\alpha = lcm(n, p)$ is the least common multiple of n and p, and $\otimes$ is the Kronecker product.

It should be pointed out that the semi-tensor product is a natural generalization of the conventional matrix product. In the sequel, we omit the symbol "$\ltimes$" if no confusion arises.

By representing the Boolean values "1" and "0" as the canonical vectors "δ_2^1" and "δ_2^2", respectively, we have $\mathcal{D} \sim \Delta$, where "$\sim$" denotes two different expressions of the same thing. In most places of this work, we use δ_2^1 and δ_2^2 to express logical variables and call them the vector form of logical variables. We have the following result on the algebraic expression of logical functions.