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A note on “Applying fuzzy linguistic preference relations to the improvement of consistency of fuzzy AHP”

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ABSTRACT

Wang and Chen (2008) stated and proved some results (Proposition 5.1, Proposition 5.2, Information Sciences, 178 (2008), 3759) and used these results to propose a method to construct fuzzy linguistic preference relation matrices. In this paper, it is pointed out that Wang and Chen have used some mathematical incorrect assumptions for proving these results. Hence, the statement and proof of these results as well as the method, proposed by Wang and Chen, are not valid. Further, the exact results are stated and proved.

1. Introduction

Wang and Chen [1, pp. 3759] stated and proved the following results.

Proposition 5.1. **[1, pp. 3759].** *Given that a set of alternatives,* $X = \{x_1, \ldots, x_n\}$ *associated with a fuzzy reciprocal multiplicative preference matrix* $\tilde{A} = (\tilde{a}_{ij})$ *with* $\tilde{a}_{ij} \in [1/9,\ 9]$, *and the corresponding fuzzy reciprocal linguistic preference relation,* $\tilde{P} = (\tilde{p}_{ij})$ *with* $\tilde{p}_{ij} \in [0, 1]$, *verifies the additive reciprocal, then, the following statements are equivalent.*

(1) $p^L_{ij} + p^R_{ji} = 1 \forall i, j \in \{1, \ldots, n\}$.
(2) $p^M_{ij} + p^M_{ji} = 1 \forall i, j \in \{1, \ldots, n\}$.
(3) $p^R_{ij} + p^L_{ji} = 1 \forall i, j \in \{1, \ldots, n\}$.

Proof. [1, pp. 3759] Since, $\tilde{A} = (\tilde{a}_{ij})$ is a reciprocal matrix,

So, $\tilde{a}_{ji} = \tilde{a}_{ij}^{-1} = \frac{1}{\tilde{a}_{ij}} \Rightarrow \tilde{a}_{ij} \otimes \tilde{a}_{ji} \approx 1$

Taking logarithms on both sides yields

$$\log_9 \tilde{a}_{ij} \oplus \log_9 \tilde{a}_{ji} \approx 0\ \forall\ i, j \in \{1, \ldots, n\}$$

Adding 2 and dividing 2 on both sides,

$$\frac{1}{2}\left(1 \oplus \log_9 \tilde{a}_{ij}\right) \oplus \frac{1}{2}(1 \oplus \log_9 \tilde{a}_{ji}) \approx \tilde{1}\ \forall i, j \in \{1, \ldots, n\}$$

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Assuming

$$\tilde{p}_{ij} = \frac{1}{2}\left(1 \oplus \log_9 \tilde{a}_{ij}\right) \text{ and } \tilde{p}_{ji} = \frac{1}{2}\left(1 \oplus \log_9 \tilde{a}_{ji}\right)$$

$$\frac{1}{2}\left(1 \oplus \log_9 \tilde{a}_{ij}\right) \oplus \frac{1}{2}(1 \oplus \log_9 \tilde{a}_{ji}) \approx \tilde{1}\ \forall i, j \in \{1, \ldots, n\}$$

$$\Rightarrow \tilde{p}_{ij} \oplus \tilde{p}_{ji} \approx \tilde{1}\ \forall i, j \in \{1, \ldots, n\}$$

$$\Rightarrow (p_{ij}^L, p_{ij}^M, p_{ij}^R) \oplus \tilde{p}_{ji} \approx \tilde{1} = (1, 1, 1)$$

$$\Rightarrow \tilde{p}_{ji} \approx 1\Theta\left(p_{ij}^L, p_{ij}^M, p_{ij}^R\right) = \left(1 - p_{ij}^R, 1 - p_{ij}^M, 1 - p_{ij}^L\right)$$

$$\left(p_{ji}^L, p_{ji}^M, p_{ji}^R\right) \approx (1 - p_{ij}^R,\ 1 - p_{ij}^M,\ 1 - p_{ij}^L$$

$$p_{ij}^L + p_{ji}^R = 1,\ \ p_{ij}^M + p_{ji}^M = 1,\ \ p_{ij}^R + p_{ji}^L = 1\ \forall\ i, j, k$$

Proposition 5.2. [1, pp. 3759] *For a reciprocal fuzzy linguistic preference relation $\tilde{P} = (\tilde{p}_{ij}) = (p_{ij}^L, p_{ij}^M, p_{ij}^R)$ to be consistent, verifies the additive consistency, then, the following statements must be equivalent:*

(a) $p_{ij}^L + p_{jk}^L + p_{ki}^R = \frac{3}{2}\ \forall\ i < j < k.$

(b) $p_{ij}^M + p_{jk}^M + p_{ki}^M = \frac{3}{2}\ \forall\ i < j < k.$

(c) $p_{ij}^R + p_{jk}^R + p_{ki}^L = \frac{3}{2}\ \forall\ i < j < k.$

Proof. [1, pp. 3759] Since, $\tilde{A} = (\tilde{a}_{ij})$ is a consistent matrix. So,

$$\tilde{a}_{ij} \otimes \tilde{a}_{jk} \cong \tilde{a}_{ik} \forall\ i, j, k.$$

Taking logarithms on both sides yields

$$\log_9 \tilde{a}_{ij} \oplus \log_9 \tilde{a}_{jk} = \log_9 \tilde{a}_{ik}\ \forall i, j, k,$$

$$\log_9 \tilde{a}_{ij} \oplus \log_9 \tilde{a}_{jk} \Theta \log_9 \tilde{a}_{ik}\ = \tilde{0},$$

$$\log_9 \tilde{a}_{ij} \oplus \log_9 \tilde{a}_{jk} \oplus \log_9 \tilde{a}_{ki}\ = \tilde{0}.$$

Adding 3 and dividing by 2 on both sides

$$\frac{1}{2}(1 \oplus \log_9 \tilde{a}_{ij}) \oplus \frac{1}{2}\ \left(1 \oplus \log_9 \tilde{a}_{jk}\right) \oplus \frac{1}{2}(1 \oplus \log_9 \tilde{a}_{ki}\ ) = \frac{\tilde{3}}{\tilde{2}}\ \ \forall i, j, k.$$

Assuming

$$\tilde{p}_{ij} = \frac{1}{2}(1 \oplus \log_9 \tilde{a}_{ij}), \tilde{p}_{jk} = \frac{1}{2}(1 \oplus \log_9 \tilde{a}_{jk}), \tilde{p}_{ki} = \frac{1}{2}(1 \oplus \log_9 \tilde{a}_{ki})$$

$$\frac{1}{2}(1 \oplus \log_9 \tilde{a}_{ij}) \oplus \frac{1}{2}\left(1 \oplus \log_9 \tilde{a}_{jk}\right) \oplus \frac{1}{2}(1 \oplus \log_9 \tilde{a}_{ki}\ ) = \frac{\tilde{3}}{\tilde{2}}\ \ \forall i, j, k.$$

$$\Rightarrow \tilde{p}_{ij} \oplus \tilde{p}_{jk} \oplus \tilde{p}_{ki} = \frac{\tilde{3}}{\tilde{2}} = \left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right)$$

$$\Rightarrow \left(p_{ij}^L, p_{ij}^M, p_{ij}^R\right) \oplus \left(p_{jk}^L, p_{jk}^M, p_{jk}^R\right) \oplus \tilde{p}_{ki} = \left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right)$$

$$\Rightarrow \tilde{p}_{ki} = \left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right)\Theta\left(p_{ij}^L + p_{jk}^L,\ \ p_{ij}^M + p_{jk}^M, p_{ij}^R + p_{jk}^R\right),$$

$$\Rightarrow \tilde{p}_{ki} = \left(p_{ki}^L,\ \ p_{ki}^M, p_{ki}^R\right) = \left(\frac{3}{2} - p_{ij}^R - p_{jk}^R,\ \ \frac{3}{2} -\ \ p_{ij}^M - p_{jk}^M, \frac{3}{2} - p_{ij}^L - p_{jk}^L\right),$$

$$\therefore\ \ p_{ij}^L +\ \ p_{jk}^L + p_{ki}^R = \frac{3}{2},\ \ p_{ij}^M +\ \ p_{jk}^M + p_{ki}^M = \frac{3}{2},\ \ p_{ij}^R +\ \ p_{jk}^R + p_{ki}^L = \frac{3}{2}.$$

Thus the expressions (a)–(c) are obtained.