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Fuzzy equivalence relation and its multigranulation spaces

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ABSTRACT

Multigranulation rough sets focus on describing uncertain sets in different granularity spaces. Different approximation accuracy of an uncertain set can be obtained through changing knowledge granularities according to user's need. However, it is a bottleneck for establishing a multigranulation space to better describe an uncertain set in a proper granulation space. And the typical way of a multigranulation space is a hierarchical quotient space determined by a fuzzy equivalence relation. In this paper, a new concept, granularity of binary relation (GB) is put forward firstly. And knowledge granulation (GK) is proved to be a special case of GB. The changing rules of granularity series for describing a hierarchical quotient space structure are proposed, and approximation accuracy series of an uncertain set is presented. For automatically searching optimal knowledge space according to user's requirement, a utility function with both knowledge space granularity and approximation accuracy is proposed. Then classification isomorphism and granularity isomorphism between two hierarchical quotient space structures are defined and discussed. Through the hierarchical quotient space structure of a fuzzy equivalence relation, the essence of classification ability of a fuzzy equivalence relation is also revealed. Finally, a new algorithm for establishing isomorphic fuzzy equivalence relations based on a hierarchical quotient space structure is presented, and this algorithm can further explain the nature of those isomorphic fuzzy equivalence relations from a new perspective.

1. Introduction

For artificial intelligence research, granular computing (GrC) is a new methodology for simulating human thinking and solving complicated problems [35,36,51,52]. It is regarded as an umbrella covering the theories, methodologies and techniques on granularity [65] and a powerful tool for solving complex problems in different granularity spaces, such as data mining, fuzzy information processing, large scale computing, cloud computing, and so on [1,9,13,22]. Two basic issues of granular computing are constructing multigranulation spaces and computing with granules in multigranulation spaces. The former focuses on information granulation, such as formation, representation and interpretation of granules, while the latter focuses on computing with granules and synthesis [48–50]. However, how to granulate a complex problem into many finer sub-problems is a bottleneck in intelligent computation.

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With the development of granular computing theory, different granular computing models have been revealed, among which there are three main approximate computing models, fuzzy sets, rough sets and quotient space. Fuzzy set [57] is an important tool to deal with uncertain problems and has been successfully applied to reality, such as fuzzy clustering, fuzzy control, fuzzy reasoning, fuzzy decision [5,17,59], and so on. Fuzzy sets describe an uncertain set with specific membership function, which embodies the uncertainty degree of an object to its set. A topological definition of fuzzy set presented by Lin [21] with neighborhood systems discusses the properties of fuzzy set from its structure. Based on fuzzy sets, fuzzy equivalence relation was presented by Zadeh [58] as a generalization of equivalence relation on a finite domain. Then fuzzy equivalence class and fuzzy partition were proposed and analyzed in detail by many scholars [2–4,6,7,30]. However, there are varied viewpoints about the meaning of a subjective concept and different membership functions for the same concept, which leads to the problem of membership degree disunity of an object from different scholars. In order to overcome this problem, Zhang and Zhang [67] put forward a structural definition of a fuzzy set based on fuzzy equivalence relation and quotient space theory [66], which uncovered some inherent properties of fuzzy set, and provided some new insights into the membership function, namely a hierarchical quotient space structure of a fuzzy equivalence relation. According to their view, a fuzzy set (or uncertain set) may have different membership degrees due to diverse membership function selection; nevertheless, there should be the same or similar characters as long as the hierarchical structures of these membership functions remain the same [63]. Zhao [68] extended quotient space theory based on weighted equivalence relations and tolerance relations and raised an effective tool for describing hierarchical structures and proposed methods for solving a complex problem with hierarchical coordinates, which are effective to determine the optimal paths of large-scale networks.

According to quotient space theory, the hierarchical quotient space structure is the essence of a fuzzy equivalence relation. And the problem of classification lies in the fact that similarity between any two objects in finite domain can come from experts. That is to say, a fuzzy similarity relation matrix can be easily found in reality. Then a fuzzy equivalence relation can be obtained based on a fuzzy similarity relation by computing its transitive closure, and a hierarchical quotient space structure can be constructed by cut-set of the fuzzy equivalence relation with many discrete thresholds. This hierarchical quotient space structure fully embodies all kinds of classification results in different granularity levels. However, for an uncertain set, it can be described in different quotient spaces (i.e., knowledge spaces) with rough sets model [32] proposed by Pawlak in 1982. It is well known that the finer the granules in rough approximation space, the higher the accuracy of an uncertain set described by upper-approximation set or lower-approximation set. In general, a hierarchical space is constructed first when solving a complicated problem, and then partial solutions are obtained in different knowledge spaces in a top-down manner and a global solution of the original problem can be composed of these partial solutions in a bottom-up manner until a satisfied solution is obtained [34,43,63].

From the viewpoint of rough sets, a partition space can be induced by an attribute subset R of an information system $IS=(U,A)$ [33], denoted by U/R. Where U is a finite domain, and R is an attribute subset of A, i.e., $R \subseteq A$. In general, on the one hand, if $\phi \neq [\,]R_1 \subset R_2 \subseteq A$, the granules in U/R_2 are finer than the granules in U/R_1, and the approximation accuracy of an uncertain set in U/R_2 is bigger than that of in U/R_1 [63]. On the other hand, if $\phi \neq R_1 \subset R_2 \subseteq A$, the attribute cost of R_2 probably is bigger than that of R_1 because the attribute values obtained from R_2 are more than that of from R_1 [62]. In the view of granular computing, Pawlak's rough sets model is based on a single granulation space, and Qian [41] extended Pawlak's model to a multigranulation rough sets model (MGRS) and involved in the evidence theory and multigranulation rough sets theory, then an information fusion approach was proposed by Lin et al. [23]. An intuitionistic fuzzy multigranulation rough sets (IFMGRS) model was established through the combination of multigranulation rough sets with intuitionistic fuzzy rough sets [15] and a test cost sensitive multigranulation rough sets model was presented by Yang et al. [46]. These models and methods enriched the rough sets theory and extended the application range. In addition, Yao and She [53] proposed a unified framework to classify, and by combining equivalence relations through set intersection and union, several extended rough sets approximation models in multigranulation spaces were established. However, the cost of establishing a multigranulation space and approximation accuracy of an uncertain set in this multigranulation space need to be integrated consideration. So we need to strike a balance between the approximation accuracy and attribute cost in real problems.

From the viewpoint of granulation, Miao [24] put forward the equivalence relation of knowledge granulation. Liang [20] proposed knowledge granulation of rough approximation space and presented a new uncertainty measure formula which was a product of roughness and knowledge granulation [20]. Yao [55] analyzed the granularity of a partition as the expected granularity of all blocks of the partition with respect to the probability distribution defined by the partition. Based on Liang's knowledge granulation formula, many incremental algorithms for attribute reduction have been gradually developed according to the monotonicity of this uncertainty measures with the changing knowledge spaces [37–41]. From another angle, a comprehensive factor based on approximation accuracy and attribute cost of an uncertain set should be considered when the uncertain set is described with lower-approximation set and upper-approximation set in a knowledge space [16,26,27,54]. In this paper, a new uncertainty measure of knowledge granulation is proposed firstly based on the fuzzy equivalence relation and its corresponding hierarchical quotient space structure. Then a utility function model for evaluating approximation quality and cost of an uncertain set is established. This model provides a new insight into searching optimal knowledge space in a hierarchical quotient space, and presents a method for automatically searching the optimal threshold. Meanwhile, many related concepts, such as classification isomorphism, granularity isomorphism, partition series and granularity series, are defined one by one. At last a new algorithm for establishing the isomorphic fuzzy equivalence relations is proposed on the basis of a given hierarchical quotient space structure.