Information Sciences 346–347 (2016) 236–260

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Knowledge reduction of dynamic covering decision information systems when varying covering cardinalities

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ARTICLE INFO

Article history:
Received 1 June 2015
Revised 1 December 2015
Accepted 27 January 2016
Available online 10 February 2016

Keywords:
Characteristic matrix
Dynamic covering approximation space
Dynamic covering decision information system
Rough set

ABSTRACT

In covering-based rough set theory, non-incremental approaches are time-consuming for performing knowledge reduction of dynamic covering decision information systems when the cardinalities of coverings change as a result of object immigration and emigration. Because computing approximations of sets is an important step for knowledge reduction of dynamic covering decision information systems, efficient approaches to calculating the second and sixth lower and upper approximations of sets using the type-1 and type-2 characteristic matrices, respectively, are essential. In this paper, we provide incremental approaches to computing the type-1 and type-2 characteristic matrices of dynamic coverings whose cardinalities vary with the immigration and emigration of objects. We also design incremental algorithms to compute the second and sixth lower and upper set approximations. Experimental results demonstrate that the incremental approaches effectively improve the efficiency of set approximation computation. Finally, we employ several examples to illustrate the feasibility of the incremental approaches for knowledge reduction of dynamic covering decision information systems when increasing the cardinalities of coverings.

1. Introduction

Since covering rough set theory was introduced by Zakowski in 1983, it has become a powerful mathematical tool for studying knowledge reduction of covering information systems [2,6–9,12,13,25,28,31–33,38,39,43,45,46,49,54,58,62–65,68,75–77]. In terms of theory, covering-based rough set theory has been combined with fuzzy sets, lattice theory, and other theories. In particular, different types of approximation operators have been proposed; their basic properties, including the relationships among them, have been investigated for knowledge reduction of covering information systems. In terms of applications, covering-based rough set theory has been employed to discover association rules from covering information systems for making decisions; the number of application domains is increasing with the development of covering-based rough set theory.

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http://dx.doi.org/10.1016/j.ins.2016.01.099
0020-0255/

Knowledge reduction of covering information systems is commonly transformed into computing approximations of sets. There are over 20 pairs of lower and upper approximation operators that are classified into different categories according to the criteria for covering approximation spaces. For example, Yao and Yao [69] classified all approximation operators into three types with respect to the interpretations of rough set approximations as follows: element-based approximation operators, granule-based approximation operators, and subsystem-based approximation operators; they are also classified into dual and non-dual operators with respect to the duality. To the best of our knowledge, the second and sixth lower and upper approximation operators in covering approximation spaces are typical examples of granule-based and element-based approximation operators, respectively, as well as dual operators, and they [17,18,30,47,61,73] have recently been attracting more attention with respect to the matrix view. For instance, Liu [30] proposed a new matrix view of rough set theory. Concretely, he presented a matrix representing an equivalence relation and redefined a pair of lower and upper approximation operators using the matrix representation in an approximation space. He also provided a fuzzy matrix representing a fuzzy equivalence relation and redefined the pair of lower and upper approximation operators for fuzzy sets using the matrix representation in a fuzzy approximation space. Wang et al. [61] presented the concepts of type-1 and type-2 characteristic matrices of coverings. They transformed the set approximation computation into products of the type-1 and type-2 characteristic matrices and the characteristic function of the set in covering approximation spaces. Zhang et al. [73] proposed the matrix characterizations of the lower and upper approximations for set-valued information systems and presented incremental approaches to updating the relation matrix to compute lower and upper approximations with dynamic attribute variation in set-valued information systems.

Incremental approaches [1,11,14–16,44,50] have emerged as effective methods for processing dynamic information, and knowledge reduction of dynamic information systems [3–5,17–24,26,27,29,34–37,40–42,48,52,53,59,60,66,67,72–74] has attracted many researchers. In practical situations, there exist a great deal of covering information systems such as incomplete information systems, and knowledge reduction of covering information systems is an important application of covering-based rough sets. Nowadays, matrix-based knowledge reduction of information systems [51,55–57,61,70] is increasingly adopted with the development of computer technology. For example, Skowron and Rauszer [51] provided a logical and theoretical foundation for knowledge reduction of information systems based on discernibility matrices. Tan et al. [55] proposed matrix-based methods for computing set approximations and reducts of covering decision information systems. Concretely, they introduced some matrices and matrix operations for computing the positive regions of covering decision information systems and employed the minimal and maximal descriptions to construct a new discernibility matrix for attribute reduction of covering information systems. Tan et al. [56] also provided fast approaches to knowledge reduction of covering information systems by employing novel matrix operations. Wang et al. [57] provided a novel method for attribute reduction of covering information systems. In particular, they developed a judgment theorem and a discernibility matrix for consistent and inconsistent covering decision systems, and presented a heuristic algorithm to find a subset of attributes to approximate a minimal reduct based on discernibility matrices. Yao and Zhao [70] proposed elementary matrix simplification operations to transform a matrix into a simpler form using heuristic reduct construction algorithms with respect to the ordering of attributes.

In practice, dynamic covering approximation spaces are time-varying. Correspondingly, dynamic covering information systems are time-varying, and the effective knowledge reduction of dynamic covering approximation spaces and covering information systems has become a significant challenge. Knowledge reduction of dynamic covering information systems is transformed into the construction of the set approximations. It is therefore necessary to investigate the computation of set approximations in dynamic covering approximation spaces. To address this, Lang et al. [17,18] presented incremental approaches to computing the second and sixth lower and upper approximations of sets in dynamic covering approximation spaces and performed knowledge reduction of dynamic covering information systems with variations of object sets. However, they focused on dynamic covering approximation spaces whose dynamic covering cardinalities remain unchanged with varying object sets. In reality, there are many dynamic covering approximation spaces whose dynamic covering cardinalities increase or decrease with the variation of object sets. Thus, computing the second and sixth lower and upper approximations of sets for knowledge reduction of dynamic covering information systems using the type-1 and type-2 characteristic matrices in the dynamic covering approximation spaces will enrich covering-based rough set theory from the matrix view.

The purpose of this paper is to investigate knowledge reduction of dynamic covering decision information systems while changing the cardinalities of coverings. First, we study the properties of dynamic coverings whose cardinalities increase and decrease because of object immigration and emigration, respectively, and present incremental approaches to computing the type-1 and type-2 characteristic matrices of dynamic coverings for the construction of set approximations. We also show four examples to demonstrate the construction of the second and sixth lower and upper approximations of sets in dynamic covering approximation spaces. Second, we provide incremental algorithms for constructing the second and sixth lower and upper approximations of sets using the type-1 and type-2 characteristic matrices, respectively. We also compare the computational complexities of the incremental algorithms with those of non-incremental algorithms. Third, we apply the incremental algorithms to large-scale dynamic covering approximation spaces and employ the experimental results to demonstrate that the incremental algorithms effectively compute the second and sixth lower and upper approximations of sets in dynamic covering approximation spaces. We show several examples to illustrate the knowledge reduction of dynamic covering decision information systems while increasing the cardinalities of coverings.

The remaining of this paper is organized as follows. Section 2 briefly reviews the basic concepts of covering-based rough set theory. In Section 3, we introduce incremental approaches to computing the type-1 and type-2 characteristic matrices