Information Sciences 370–371 (2016) 123–126

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Comment on "An algorithm for solving optimization problems with fuzzy relational inequality constraints"

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ARTICLE INFO

Article history:
Received 19 April 2015
Revised 4 July 2016
Accepted 21 July 2016
Available online 28 July 2016

Keywords:
Linear programming
Fuzzy relational inequality

ABSTRACT

A counterexample is provided to illustrate the incorrectness of I^0 in Rule 1 presented by Guo et al. [F.F. Guo, L.P. Pang, D. Meng, Z.Q. Xia. An algorithm for solving optimization problems with fuzzy relational inequality constraints, Information Sciences 252 (2013) 20-31]. In other words, (7b) and ($7b^*$) in the referenced paper above are not equivalent unless index set I^0 is revised, since the definition of I^0 in Rule 1 can only be correct when the constraints are fuzzy relational equations. And finally the correct of I^0 is also presented in this note.

1. Introduction

The definition of I^0 in Rule 1 given by Guo et al. [1] can only be correct correct when the constraints are fuzzy relational equations. It follows that (7b) and ($7b^*$) in Ref. [1] are not equivalent unless index set I^0 is revised. A counterexample is given below for the case of fuzzy relational inequality constraints.

Example. Consider the following problem,

$$\begin{aligned} \min \quad & f(x) = 3x_1 + 4x_2 - x_3 + x_4 + 2x_5 - 5x_6 \\ \text{s.t.} \quad & A \circ x \geqslant b, \\ & B \circ x \leqslant d, \\ & x \in [0,1]^n \end{aligned} \tag{1}$$

where

$$A = \begin{bmatrix} 0.5 & 0.8 & 0.9 & 0.3 & 0.85 & 0.4 \\ 0.2 & 0.2 & 0.1 & 0.95 & 0.1 & 0.8 \\ 0.8 & 0.8 & 0.4 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4 & 0.7 & 0.95 & 0.4 & 0.9 & 0.5 \\ 0.3 & 0.3 & 0.2 & 1.0 & 0.2 & 0.85 \\ 0.8 & 0.75 & 0.3 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.0 & 0.0 & 0.2 & 0.0 & 0.0 \end{bmatrix},$$
$$b = [0.85\ 0.6\ 0.5\ 0.1]^T, \quad d = [0.9\ 0.8\ 0.7\ 0.2]^T.$$

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http://dx.doi.org/10.1016/j.ins.2016.07.050
0020-0255/

Step 1. Compute the maximum feasible solution $\bar{x}$ by, for any $j \in \{1, 2, 3, 4, 5, 6\}$,

$$\bar{x}_j = \begin{cases} \min\{d_i | b_{ij} > d_i, i \in L\}, & \text{if } \{d_i | b_{ij} > d_i, i \in L\} \neq \emptyset, \\ 1, & \text{if } \{d_i | b_{ij} > d_i, i \in L\} = \emptyset. \end{cases}$$

We have $\bar{x} = (0.7\ 0.7\ 0.9\ 0.8\ 1.0\ 0.8)^T$ and $A \circ \bar{x} \geqslant b$, then go to Step 2.

Step 2. Obviously, $J^0 = \{3, 6\}$, $I^0 = \{4\}$ on the basis of $J^0 = \{j \in J | c_j \leqslant 0\}$ and $I^0 = \{i \in I | \exists j \in J^0 \text{ such that } \min\{a_{ij}, \bar{x}_j\} = b_i\}$. By Rule 1, we can set $x_3^* = \bar{x}_3 = 0.9, x_6^* = \bar{x}_6 = 0.8$, and delete columns 3 and 6 of A and eliminate the fourth constraint of $A \circ x \geq b$. So, $\bar{I} = \{1, 2, 3\}, \bar{J} = \{1, 2, 4, 5\}$. Then, problem (1) is converted into the following one,

$$\begin{array}{ll} \min & Z = z(x) = 3x_1 + 4x_2 + x_4 + 2x_5 - 4.9 \\ \text{s.t.} & A' \circ x \geqslant b', \\ & 0 \leqslant x_i \leqslant \bar{x}_j, \quad j \in \bar{J}, \end{array} \tag{2}$$

where $b' = [0.85\ 0.6\ 0.5]^T$ and

$$A' = \begin{array}{c} i\backslash j \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 4 & 5 \end{array} \\ \begin{bmatrix} 0.5 & 0.8 & 0.3 & 0.85 \\ 0.2 & 0.2 & 0.95 & 0.1 \\ 0.8 & 0.8 & 0.1 & 0.1 \end{bmatrix} \end{array}.$$

By virtue of $I_j := \{i \in \bar{I} | \min\{a_{ij}, \bar{x}_j\} \geqslant b_i\}$ and $J_i := \{j \in \bar{J} | \min\{a_{ij}, \bar{x}_j\} \geqslant b_i\}$, we calculate index sets $I_j (j \in \bar{J})$ and $J_i (i \in \bar{I})$ as follows:

$$\begin{array}{l} I_1 = \{3\}, I_2 = \{3\}, I_4 = \{2\}, I_5 = \{1\}, \\ J_1 = \{5\}, J_2 = \{4\}, J_3 = \{1, 2\}. \end{array}$$

Step 3. Rule 2 cannot be applied. Due to Rule 3 and $c_1 < c_2, I_1 = I_2$, set $x_2^* = 0$ and reduce the constraints by removing the second column of A'.

Step 4. We can generate an optimal solution $x^* = [0.5\ 0.0\ 0.9\ 0.6\ 0.85\ 0.8]^T$ directly with its optimal value being $f(x^*) = -1.1$.

However, $x^0 = [0.5\ 0.0\ 0.9\ 0.0\ 0.85\ 0.8]^T$ is a feasible solution to problem (1) and $f(x^0) = -1.7 < f(x^*)$.

Why? In Step 2, we should delete constrains 1 and 2 since these in $A \circ x \geq b$ have been satisfied actually by setting $x_3 = 0.9$ and $x_6 = 0.8$ and need not to be satisfied again by setting $x_4 = 0.6$ and $x_5 = 0.85$. That's the reason why a wrong optional solution is got. Why don't we delete the constraints 1 and 2? The main reason is because of the definition of $I^0 = \{i \in I | \exists j \in J^0 \text{ such that } \min\{a_{ij}, \bar{x}_j\} = b_i\}$. If the constraints are fuzzy relational equations $A \circ x = b$, the definition of I^0 is correct. We first consider a problem below,

$$\begin{array}{ll} \min & Z = z(x) = 3x_1 + 4x_2 + x_4 + 2x_5 - 4.9 \\ \text{s.t.} & A \circ x \geqslant b, \\ & 0 \leqslant x_i \leqslant \bar{x}_j, \quad j \in J, \end{array} \tag{3}$$

where A and b are the same as in (1). Obviously one of minimal solutions $\underline{x}$ is an optimal one to (3). From Theorem 3 of [1], there must exist an FRI path p such that $\underline{x} = x^p$, where x^p is denoted by (5) of [1]. Compute index sets $J_i (i \in I)$ by $J_i := \{j \in J | \min\{a_{ij}, \bar{x}_j\} \geq b_i\}$,

$$J_1 = \{3, 5\}, J_2 = \{4, 6\}, J_3 = \{1, 2\}, J_4 = \{1, 2, 3, 4, 5\}.$$

By Rule 2 of [1] and $J_1 \subseteq J_4$, it has no effect on the optimal solution of (3) by deleting J_4. In order to get an optimal solution of (3), we must choose $p_1 = 3$ and $p_2 = 6$ from J_1 and J_2, since $c_3 = c_6 = 0$ and $c_4 > 0$, $c_5 > 0$. Due to Rule 3 and $c_1 < c_2, I_1 = I_2$, set $p_3 = 1$. Thus, $x^* = x^p = [0.5\ 0.0\ 0.85\ 0.0\ 0.0\ 0.6]^T$. It follows from Theorem 3 of [1] that an optimal solution to (1) is $x^* = [0.5\ 0.0\ 0.9\ 0.0\ 0.0\ 0.8]^T$.

From the discussion above, we see that it has no effect on the optimal solution to (3) by removing J_1 and J_2 or 1 and 2 constraints of problem (1). However, we can not delete constraints 1 and 2 by Rule 1 since $a_{13} \wedge \bar{x}_3 \neq b_1, a_{16} \wedge \bar{x}_6 \neq b_1, a_{23} \wedge \bar{x}_3 \neq b_2, a_{26} \wedge \bar{x}_6 \neq b_2$, that is, $1, 2 \notin I^0$.