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Selection mechanisms based on the maximin fitness function to solve multi-objective optimization problems

Adriana Menchaca-Mendez*, Carlos A. Coello Coello

CINVESTAV-IPN, Departamento de Computación Av. IPN 2508 San Pedro Zacatenco México D.F. 07300, México

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ABSTRACT

In this paper, we study three selection mechanisms based on the maximin fitness function and we propose another one. These selection mechanisms give rise to the following MOEAs: “MC-MOEA”, “MD-MOEA”, “MH-MOEA” and “MAH-MOEA”. We validated them using standard test functions taken from the specialized literature, having from three up to ten objective functions. We compare these four MOEAs among them and also with respect to MOEA/D (which is based on decomposition), and to SMS-EMOA (which is based on the hypervolume indicator). Our preliminary results indicate that “MD-MOEA” and “MAH-MOEA” are promising alternatives for solving MOPs with either low or high dimensionality.

1. Introduction

In real-world applications there are many problems which involve the simultaneous optimization of multiple objective functions [1], which are normally in conflict with each other. They are called “*Multi-objective Optimization Problems (MOPs)*”. Because of the conflicting nature of the objectives to be optimized, the notion of optimality refers in this case to finding the best possible trade-offs among the objectives (i.e., we aim to find solutions for which no objective can be improved without worsening another). Consequently, when solving MOPs we do not aim to find a single optimal solution but a set of them, which constitute the so-called Pareto optimal set, whose image is known as the Pareto front.

The use of evolutionary algorithms for solving MOPs has become very popular in the last few years [2], giving rise to the so-called Multi-Objective Evolutionary Algorithms (MOEAs).[1] MOEAs have two main goals: (i) to find solutions that are, as close as possible, to the true Pareto front and, (ii) to produce solutions that are spread along the Pareto front as uniformly as possible. We can talk of two types of MOEAs, if we classify them based on their selection mechanism: (i) those that incorporate the concept of Pareto optimality into their selection mechanism, and (ii) those that do not use Pareto dominance to select individuals. The use of Pareto-based selection has several limitations from which, its poor scalability with respect to the number of objective functions is, perhaps, the most remarkable. The quick increase in the number of non-dominated solutions as we increase the number of objective functions, rapidly dilutes the effect of the selection mechanism of a MOEA [9].

Here, we are interested in the maximin fitness function (MFF) [10,11] which can act as a selection mechanism of type (ii) and it has interesting properties. Furthermore, computing the MFF is computationally efficient because its complexity is linear with

* Corresponding author. Tel.: +1 55 57532389.
E-mail addresses: adriana.menchacamendez@gmail.com (A. Menchaca-Mendez), ccoello@cs.cinvestav.mx (C.A. Coello Coello).

[1] Although this paper focuses on MOEAs, there are many other multi-objective metaheuristics currently available (for example, multi-objective ant colony optimizers [3,4], multi-objective particle swarm optimizers [5], multi-objective firefly algorithms [6], multi-objective flower pollination algorithms [7], and multi-objective harmony search algorithms [8] just to name a few). However, their discussion is beyond the scope of this paper.

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respect to the number of objective functions. Nevertheless, the use of the MFF also has some disadvantages, but there have been some proposals to address them.

Thus, the main goal of this paper is to provide an in-depth study about the MFF and its proposed variations, so that we can identify its main advantages and possible limitations. Such a study aims to provide more information about the sort of instances in which it is advisable to use any of the proposed MFF-based MOEAs, as well as those cases in which their use may present some difficulties.

The remainder of this paper is organized as follows. Section 2 states the problem of our interest. The previous related work about MOEAs based on MFF is presented in Section 3. MFF is described in detail in Section 4. Section 5 describes three MOEAs based on MFF (MC-MOEA, MD-MOEA and MH-MOEA) and we also propose a new version of MH-MOEA called MAH-MOEA. Our experimental results are presented in Section 6. Finally, we provide our conclusions and some possible paths for future work in Section 7.

2. Problem statement

We are interested in the general *multiobjective optimization problem (MOP)*, which is defined as follows: Find $\vec{x}^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$ which optimizes

$$\vec{f}(\vec{x}) = [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})]^T \tag{1}$$

such that $\vec{x}^* \in \Omega$, where $\Omega \subset R^n$ defines the feasible region of the problem. Assuming minimization problems, we have the following definitions.

Definition 1. We say that a vector $\vec{x} = [x_1, \ldots, x_n]^T$ dominates vector $\vec{y} = [y_1, \ldots, y_n]^T$, denoted by $\vec{x} \prec \vec{y}$, if and only if $f_i(\vec{x}) \leq f_i(\vec{y})$ for all $i \in \{1, \ldots, k\}$ and there exists an $i \in \{1, \ldots, k\}$ such that $f_i(\vec{x}) < f_i(\vec{y})$.

Definition 2. We say that a vector $\vec{x} = [x_1, \ldots, x_n]^T$ is weakly non-dominated if there does not exist any $\vec{y}$ such that $f_i(\vec{y}) < f_i(\vec{x})$ for all $i \in \{1, ..., k\}$.

Definition 3. A point $\vec{x}^* \in \Omega$ is Pareto optimal if there does not exist any $\vec{x} \in \Omega$ such that $\vec{x} \prec \vec{x}^*$.

Definition 4. A point $\vec{x} \in \Omega$ is weakly Pareto optimal if there does not exist another point $\vec{y} \in \Omega$ such that $f_i(\vec{y}) < f_i(\vec{x})$ for all $i \in \{1, ..., k\}$.

Definition 5. For a given MOP, $\vec{f}(\vec{x})$, the Pareto optimal set is defined as: $\mathcal{P}^* = \{\vec{x} \in \Omega | \neg\exists \vec{y} \in \Omega : \vec{y} \prec \vec{x}\}$.

Definition 6. Let $\vec{f}(\vec{x})$ be a given MOP and $\mathcal{P}^*$ the Pareto optimal set. Then, the Pareto Front is defined as: $\mathcal{PF}^* = \{\vec{f}(\vec{x}) \mid \vec{x} \in \mathcal{P}^*\}$.

3. Related work

The maximin fitness function (MFF) was originally proposed by Balling in [10] and it has been incorporated in Genetic Algorithms [11–15], particle swarm optimizers [16,17], ant colony optimizers [18] and differential evolution [19].

The early proposals based on MFF only considered MOPs with low dimensionality (two objective functions) and did not adopt a technique to improve the distribution based on the idea that MFF penalizes clustering. It was until 2012 [19] that a more in-depth study of MOEAs based on MFF was undertaken. The authors of this study found two important disadvantages when MFF is used to select individuals:

1. MFF prefers weakly non-dominated individuals over dominated individuals and this causes a loss in the diversity of the population, especially, in MOPs in which one objective function is easier to solve than the others.
2. The second disadvantage has to do with the poor diversity obtained in objective function space. Although MFF penalizes clustering between solutions, it is possible that many individuals have the same fitness and then we cannot know which individual should be selected.

In recent years, some proposals to address the two above disadvantages have been made [13–15,19]. In the following sections we will provide an in-depth analysis of such proposals.

4. Maximin fitness function

The maximin fitness function (MFF) works as follows. Let's consider a MOP with K objective functions and an evolutionary algorithm whose population size is P. Let f_k^i be the normalized value of the kth objective for the ith individual in a particular generation. Assuming minimization problems, we have that the jth individual weakly dominates the ith individual if: $min_k(f_k^i - f_k^j) \geq 0$. The ith individual, in a particular generation, will be weakly dominated by another individual, in the generation, if: $max_{j \neq i}(min_k(f_k^i - f_k^j)) \geq 0$. Then, the maximin fitness of individual i is defined as:

$$fitness^i = max_{j \neq i}(min_k(f_k^i - f_k^j)) \tag{2}$$