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Some consistency measures of extended hesitant fuzzy linguistic preference relations

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A B S T R A C T

Linguistic preference relations (LPRs) enable decision makers to express preferences by pair-wise comparisons in qualitative setting. The fundamental aspect of LPRs is to measure the degrees of consistency when applying them in decision making. Extended hesitant fuzzy linguistic term sets (EHFLTSs) are a powerful tool for modeling uncertain linguistic information in group decision making. Based on which we first present the concept of extended hesitant fuzzy linguistic preference relations (EHFLPRs), and then develop the additive consistency measure and the weak consistency measure respectively. Furthermore, in order to solve the consistency problem by the perspective of graphs, two algorithms are proposed based on two kinds of predefined graphs, i.e., the hesitant preference graphs and the symmetric hesitant preference graph. The selective algorithm selects the arcs with the highest additive consistency level from the symmetric hesitant preference graph and constructs a LPR. While the broken circle algorithm removes the arcs from circular triads of the hesitant preference graph, it makes the EHFLPR with weak consistency and divides the EHFLPR into several possible LPRs satisfying weak consistency. This paper therefore explores a visible interpretation of consistency measures. The rationality of the proposed algorithms is verified by several examples, and some related issues are also discussed.

1. Introduction

Decision making by a group of decision makers (DMs), such as committees, governing bodies, juries, business partners, teams, and families, is referred to as group decision making. Along with the increasing complexity of practical problems, it is very convenient for the DMs to provide evaluations by pair-wise comparisons. Such comparison information can be contained in a preference relation, such as multiplicative preference relations (MPRs) [36], fuzzy preference relations (FPRs) [13,55] and linguistic preference relations (LPRs) [14,15,20,62]. LPRs, associated with the corresponding techniques of computing with words (CWW) [58,60], are commonly used in qualitative cases where it is difficult to conduct evaluations by means of precise and exact values [29].

Based on classical linguistic term sets [21] and virtual linguistic term sets [51], multiplicative LPRs [46,49], additive LPRs [14,48,50], distribution-based LPRs [62] and incomplete LPRs [8,44] have been investigated widely over the last decades. With the uncertain linguistic terms [51], uncertain LPRs [9,31,53,64–66] have been discussed. Additionally, on the basis

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of linguistic 2-tuple representation model [22] or intuitionistic fuzzy sets [2], the 2-tuple LPRs [12,18] and the 2-tuple intuitionistic fuzzy LPRs [63] have been proposed respectively. Motivated by the idea of hesitant fuzzy sets (HFSs) [39], Rodriguez et al. [35] proposed the hesitant fuzzy linguistic term sets (HFLTSs), which were further investigated in Refs. [24,26,43,45]. Recently, Zhu and Xu [68] and Liu et al. [25] studied hesitant fuzzy linguistic preference relations (HFLPRs) in the context of HFLTSs. Wang [42] extended the HFLTSs into group decision making situations, and proposed the concept of extended hesitant fuzzy linguistic term sets, which can also be called typical extended hesitant fuzzy linguistic term sets as recommended by Bedregal et al. [3]. An EHFLTS is a discrete or consecutive subset of a given linguistic term set and thus is more flexible to manage a group's assessments under uncertainties. The main purpose of this paper is to investigate preference relations taking the form of EHFLTSs.

Consistency measures of preference relations are the vital basis of group decision making and have been studied extensively. Numerous studies have focused on MPRs [1,37] and FPRs [19] whose entries are real numbers and fuzzy numbers respectively. Motivated by those of MPRs and FPRs, most of the consistency measures of LPRs have concentrated on the predefined transitivity, such as the additive transitivity and the weak transitivity. According to Kwiesielewicz and Uden [23], there are contradictory judgments even if a preference relation passes the test of consistency. Thus acceptable consistency is, in most cases, a rational alternative when strict consistency is hard to reach, and the LPRs with acceptable consistency were studied in Refs. [9,14,65,66,68]. To provide a minimum condition of a logically consistent preference relation, the weak consistency [38,47] based on weak transitivity has been commonly taken into account. Moreover, graph theory is a popular tool to deal with weak consistency [17,27,47,67].

However, there are some limitations in the existing relevant studies. Firstly, the basic operations defined in Ref. [68] do not match the classical operations on HFSs. Thus there always exists a normalization step, such as β-normalization, whenever the operation "addition" is used. The normalization step may bring some new values which are not included in the original information. The first motivation of this paper is therefore to redefine some basic concepts which generalize those of Ref. [68] and adhere to the idea of HFSs. Secondly, although Zhu and Xu [68] presented a flexible consistency testing method to identify whether a HFLPR is of acceptable consistency, it is usually difficult to obtain high acceptable consistency due to the limitation of human capability in comparing a large number of alternatives. Therefore, it is necessary to introduce the weak consistency for HFLPRs to ensure that they are logically correct. Thirdly, if a HFLPR is acceptable according to Ref. [68], there is no technique to explore its priorities. Because each element involved in an EHFLTS implies a possible exact value, it is reasonable to explore the possible LPRs with the highest consistency level or with weak consistency from the original HFLPRs. When the reduced LPR is derived, the consistency checking and the ranking can be obtained by the existing techniques for the LPR, such as the methods proposed by Xu et al. [48]. No additional ranking technique is needed, and finally, the existing studies use little information of the digraph of preference relations. We will demonstrate that if the weights of digraph are well defined and used, then both the additive consistency and the weak consistency can be visually interpreted by the perspective of graph theory [4].

In this paper, we discuss the additive consistency and the weak consistency of LPRs whose entries are EHFLTSs. When expressing preference information among alternatives in quality setting, the DMs in a group may hesitate about some possible linguistic terms. Therefore, with the EHFLTSs and their equivalence relation, we first develop a new type of preference relations (i.e., EHFLPRs), and then discuss the additive consistency measure and the weak consistency measure. Based on which we explore the possible LPRs with the highest consistency level or with weak consistency from the original EHFLPRs.

The rest of the paper is organized as follows: In Section 2, we review the linguistic representation models, and the concepts of HFLTSs, EHFLTSs and EHFLPRs. Section 3 introduces some specific concepts of digraphs used in this paper. The additive consistency measure and the weak consistency measure, as well as the corresponding algorithms, are presented in Sections 4 and 5 respectively. Section 6 makes some discussions, and finally, concluding remarks are given in Section 7.

2. EHFLTSs and EHFLPRs

In this section, we start by introducing the linguistic representation models and some operations on EHFLTSs.

2.1. Linguistic representation models

In many real-world situations, the use of linguistic information is very straightforward and suitable to express the satisfaction associated with an outcome and a state of nature [28,30,35,40,54]. Fuzzy linguistic approaches [57–59] have been employed to model the linguistic information, and the fuzzy set theory [61] has been utilized to manage uncertainties. In Ref. [51], Xu defined the linguistic term set as:

$$S = \{s_\alpha | \alpha = -t, \ldots, t\}, \tag{1}$$

where t is a positive integer such that the cardinality of S should be neither too small nor too rich [5], and s_α owns the following characteristics:

(1) The set is ordered: $s_i \geqslant s_j$ iff $i \geqslant j$;
(2) The negation operator is defined: $neg(s_\alpha) = s_{-\alpha}$, especially $neg(s_0) = s_0$.