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On reaching group consensus for linearly coupled multi-agent networks

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ABSTRACT

In this paper, the problems of group consensus for linearly coupled multi-agent networks including first-order and second-order are investigated, respectively. Based on the Laplacian matrix associated with the weighted adjacency matrix of the system, we present two novel linear protocols which can exactly reflect the interactive influence between the agents of the multi-agent network. Instead of relying on other conservative assumptions presented by the majority of the relevant research works, some promising criteria which can guarantee the reaching of group consensus of the multi-agent network are also obtained analytically. In addition, we also extend our work to study the group consensus for the multi-agent network with generally connected topology which neither needs to be strongly connected nor needs to contain a directed spanning tree. The conclusion that we have obtained should be more representative. Finally, the validity and correctness of our theoretical results are verified by several numerical simulated examples.

1. Introduction

Consensus is a typical problem for distributed coordinative control of multi-agent networks. It aims to design some appropriate protocols and controllable strategies so that all agents in a network can converge to consistent states. Recently, consensus has received considerable attention due to its broad applications in biological systems, robotics teams, sensor networks, unmanned air vehicle formations, under-water vehicles, and group decision making problems, etc. Up to now, focusing on multi-agent networks, many consensus problems have been investigated deeply and many interesting research findings have been obtained simultaneously, as seen in Refs. [1–3,5,7,8,12,13,15,17,21,22,25] and the reference therein.

In order to ensure the cooperative task to be accomplished, all the agents in a group must be in agreement no matter what changes took place. This requires that all the agents are capable of sensing and responding to some unanticipated situations or any changes. As a result, a common phenomenon may happen: the agreements become different along with the changes of environments, situations, cooperative tasks or even time. Moreover, different cooperative task among multiple agents in collaborative control also may result in different agreements. This is called group (or cluster) consensus problem in multi-agent networks. Indeed, group consensus of multi-agent networks is not only the major dynamic behavior of complex networks, but also the fundamental problem of distributed coordinative control of complex systems. When group consensus is

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achieved, the agents in a network can reach more than one consistent values, i.e., the agents in the same group can reach a consistent state while there is no consensus among different sub-groups [26]. Accordingly, we can easily find that consensus problem can be regarded as a special case of group consensus. Meanwhile, group consensus can provide needed flexibility for some applications along with the increasing of the scale and complexity of complex systems.

Recently, great deals of excellent research results about group consensus have emerged constantly. Yu and Wang [26] studied the group consensus problem for the system with strongly connected and balanced digraph. And subsequently, they extended their works to undirected system and obtained some criteria guaranteeing the system to realize group average-consensus [27]. In Ref. [28], Yu and Wang also investigated the group consensus of the multi-agent system with switching topology and communication delays by using the method of double-tree-form, then some useful criteria were developed analytically. Yi et al. [24] discussed the cluster consensus of the linearly coupled systems with directed topology and revealed the relationship between the cluster numbers and the Laplacian matrix, that is the number of clusters equals to the multiplicity of the zero eigenvalue of the Laplacian. In Ref. [20], Tan et al. discussed the group consensus problems about the system with fixed and switching topology, and some necessary and/or sufficient conditions under some mild assumptions were derived. Hu et al. [6] investigated the group average-consensus in undirected multi-agent system with novel hybrid protocol, and some algebraic criteria to guarantee the group consensus of the system were established. In Ref. [10], Ji et al. considered the group consensus for two kinds of delayed systems, then they developed some less conservative criteria based on the frequency-domain analysis and generalize Nyquist criterion, respectively. Feng et al. [4] discussed the static and time-varying group consensus problems for second-order dynamic multi-agent system. In order to reduce the cost of control, pinning strategies have been introduced to the distributed control of multi-agent networks. Recently, lots of relevant research results about group consensus or cluster synchronization have been reported, such as [9,11,14,19,23] and the references therein.

Here, it is worth noting that almost all of the aforementioned works are based on the conservative assumption which is in-degree balance. This assumption requires the sum of adjacent weights from every node in one group to all nodes in the other group is equal to zero at any time. It means that there is no communication between any two sub-groups of the whole multi-agent network. The limitation of in-degree balance has been listed in details in Ref. [20]. As this assumption is too strong, Tan et al. [20] made the effort to relax this conservative condition, and their work is based on a relatively loose condition: the sum of adjacency weights from every node in one group to all nodes in another group is identical at every time. This assumption implies that the influence from every agent in one group to all agents in the other groups is always equal at any time. Compared with the condition of in-degree balance, the assumption proposed by Tan seems to be not so strong. However, both of these two assumptions are too limited in practice to investigate the common aspect of the group consensus problem. The condition which they imposed on the coupling weights between the nodes of the multi-agent network is very restrictive. In the general case, the influences among the sub-groups do exist while these influences are uncertain. Given the fact that they cannot meet the general requirements, the two aforementioned assumptions are only two special cases, at most. In this paper, we relax both of these two assumptions and investigate the group consensus problem for the first-order and second-order dynamic multi-agent systems, respectively. Based on the Laplacian matrix of the systems, two novel linear protocols are proposed and some positive criteria are developed analytically. At the same time, we also extend our work to investigate the group-consensus for the multi-agent network with generally connected topology, that is, the corresponding graph generated by the weighted adjacency matrix and the subgraph of each group can be directed, weakly connected and even there is no rooted directed spanning tree. It is also worth pointing out that some of the aforementioned works have only investigated the group consensus of multi-agent networks with special topology, such as undirected, strongly connected and containing a directed spanning tree. Compared with the existing results, our findings are more promising in a broad range of practical applications.

The rest of the paper is outlined as follows. In Section 2, some preliminaries and relevant problems are formulated. The main results are established in Section 3. And in Section 4, some numerical simulation examples are provided to validate our theoretical results, and conclusions are finally drawn in Section 5.

Notation. Throughout this paper, let R and C denote the sets of real and complex numbers, respectively. For $z \in C$, $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ represent its real and imaginary part, respectively. Let $I_n(0_n)$ be an n-dimensional identity (zero) matrix, $1_n \in R^n(0_n \in R^n)$ be the vector with all entries being 1(0). For a real symmetric matrix $A \in R^{n\times n}$, denote $A > 0(A < 0)$, if A is positive (negative) definite. Specially, denote $A \geqslant 0(A \leqslant 0)$, if A is positive (negative) semi-definite.

2. Preliminaries and problem statement

2.1. Relevant graph theory

In a multi-agent network, suppose the information exchanged among agents can be modeled by an interaction digraph and each agent can be regarded as a node in the digraph. Each agent updates its current state based on the information received from its neighbors. Let $G = \{V, \varepsilon, \mathrm{A}\}$ be a digraph (or weighted directed graph), where $V = \{v_1, \ldots, v_N\}$ is the node set and the node indexes belong to a finite index set $\ell = \{1, 2, \ldots, N\}$; $\varepsilon \subseteq V \times V$ denotes the edge set, $\mathrm{A} = (a_{ij}) \in R^{N\times N}$ is the weighted adjacency matrix which can represent the coupling configuration of the network, and the elements in matrix