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Multi-attribute decision analysis under a linguistic hesitant fuzzy environment

Fanyong Meng [a,b,*], Xiaohong Chen [a], Qiang Zhang [c]

[a] School of Business, Central South University, Changsha 410083, China
[b] School of Management, Qingdao Technological University, Qingdao 266520, China
[c] Department of Management and Economics, Beijing Institute of Technology, Beijing 100081, China

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ABSTRACT

In this paper, a new class of fuzzy sets called linguistic hesitant fuzzy sets (LHFSs) is defined, which can address the qualitative preferences of experts as well as reflect their hesitancy, uncertainty and inconsistency. Based on the defined operational laws of LHFSs and the order relationship, two linguistic hesitant fuzzy hybrid aggregation operators are defined: the generalized linguistic hesitant fuzzy hybrid weighted averaging (GLHFHWA) operator and the generalized linguistic hesitant fuzzy hybrid geometric mean (GLHFHGM) operator. Furthermore, to address the situation in which the elements in a set are interdependent, the generalized linguistic hesitant fuzzy hybrid Shapley weighted averaging (GLHFHSWA) operator and the generalized linguistic hesitant fuzzy hybrid Shapley geometric mean (GLHFHSGM) operator are presented, which are extensions of the GLHFHWA and GLHFHGM operators. Models designed to obtain the optimal fuzzy measures and additive measures on an attribute set and on an ordered set are, respectively, constructed. Then, an approach to linguistic hesitant fuzzy multi-attribute decision analysis is developed. Finally, two numerical examples are provided to demonstrate the practicality and efficiency of the proposed procedure.

1. Introduction

A multi-attribute decision-making process usually involves the following six steps [3,7,14,16,18,28,29,39,48]: (1) identify the decision-making problem; (2) establish the attribute set; (3) assess the alternative values with respect to attributes; (4) calculate their comprehensive values; (5) rank alternatives; and (6) select the best choice(s). Multi-attribute decision making, a widely practiced human activity, has been studied in depth by many researchers and widely applied in practical problems such as education [3], medical care [23], military [37], engineering [25], social sciences [5], and economics [47]. Because of the complexity of the socioeconomic environment, it is impractical for decision makers to give the exact values of alternatives to every attribute. In this situation, the decision makers must refer to fuzzy preferences that can generally be modeled quantitatively or qualitatively. Quantitative fuzzy preferences are usually expressed by fuzzy sets [71], type-2 fuzzy sets [72], (interval-valued) intuitionistic fuzzy sets [1,2], and (interval-valued) hesitant fuzzy sets [6,45]; qualitative fuzzy preferences are usually expressed by linguistic variables [73–75], such as “fast”, “fair”, or “slow”.

Linguistic variables are an effective tool for addressing problems that are too complex or too ill-defined to use quantitative expressions. These variables have been studied in depth by many scholars [8,9,14–16,20,28,32,35,50–52,57,58,63] and

* Corresponding author at: School of Business, Central South University, Changsha 410083, China. Tel.: +86 18254298903.
E-mail addresses: mengfanyongtjie@163.com (F. Meng), cxh@csu.edu.cn (X. Chen), qiangzhang@bit.edu.cn (Q. Zhang).

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are widely used in many fields [7,18,19,22,29,35,59,60]. More recently, some scholars [48] have noticed that the linguistic variable cannot reflect the membership and non-membership degrees of an element to a particular concept. Based on intuitionistic fuzzy sets, Wang and Li [48] defined intuitionistic linguistic sets (ILSs). Following the pioneer work of [48], some extensions have been presented, such as intuitionistic uncertain linguistic sets (IULSs) [26] and interval-valued intuitionistic uncertain linguistic sets (IVIULSs) [27]. As Torra noted [45], when experts make decisions, they are usually hesitant and irresolute, which makes a final agreement difficult to reach. As a result, the difficulty of establishing the membership degree does not arise because there is a margin of error or some possibility distribution on the possibility values but because there are several possible values. Recently, Rodríguez et al. [38,39] introduced hesitant fuzzy linguistic term sets (HFLTSs) that permit a linguistic variable to have several linguistic terms. HFLTSs can address the hesitancy and inconsistency of experts. However, similarly to linguistic variables, HFLTSs cannot reflect the membership degree of an element to a concrete concept.

This paper defines a new kind of linguistic variables, named linguistic hesitant fuzzy sets (LHFSs) that reflect the inconsistency, hesitancy and uncertainty of experts. Some operational laws on LHFSs are defined and an order relationship is introduced. Then, some hybrid aggregation operators that consider the importance of the elements in a set and the ordered positions are defined. To address the situation in which the weight information is completely unknown or partly known, several models are built to obtain the weight vectors. As a series of developments, we further research multi-attribute decision-making problems with qualitative attributes under the linguistic hesitant fuzzy environment.

This paper is organized as follows: Section 2 briefly reviews several types of fuzzy sets, such as HFSs, linguistic variables, and HFLTSs. Section 3 first introduces LHFSs. Then, some operational laws are defined and an order relationship is presented. Section 4 defines some hybrid aggregation operators, such as the GLHFHSWA, GLHFHSGM, GLHFHWA, and GLHFHGM operators, and some important cases are studied. Section 5 establishes linear programming models by which the optimal fuzzy measures and additive measures can be obtained. Then, an approach to linguistic hesitant fuzzy multi-attribute decision making that considers the interactions between the attributes and between the ordered positions is developed. Section 6 provides two numerical examples to demonstrate the feasibility and efficiency of the proposed procedure. Conclusions are made in the last section.

2. Basic concepts

Before introducing LHFSs, this section reviews some related concepts, such as hesitant fuzzy sets and hesitant fuzzy linguistic term sets. These concepts help us better understand LHFSs.

2.1. Hesitant fuzzy sets

As an extension of Zadeh's fuzzy sets, HFSs [45] permit the membership to be a set of possible values that can address inherent hesitancy and uncertainty in the human decision-making process. More recently, Xia and Xu [55] introduced hesitant fuzzy elements (HFEs). Inspired by the relationship between HFEs and Atanassov's intuitionistic fuzzy values (AIFVs), Xia and Xu [55] defined some operations on HFEs, and some hesitant fuzzy aggregation operators can be observed in the literature [4,11,36,53,56,66,69,70].

Definition 1 [45]. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set. A hesitant fuzzy set (HFS) in X is expressed in terms of a function such that when applied to X it returns a subset of $[0,1]$, denoted by $E = (\langle x_i, h_E(x_i)\rangle \mid x_i \in X)$, where $h_E(x_i)$ is a set of some values in $[0,1]$ denoting the possible membership degrees of the element $x_i \in X$ to the set E.

For convenience, Xia and Xu [55] called $h = h_E(x_i)$ a hesitant fuzzy element (HFE). Take the quantitative attribute of the power consumption of refrigerators as an example; because the power consumption is influenced by many factors, such as external temperature, volume size, quantity of storage items and switching frequency, different users may produce different power consumption values. Thus, it is inappropriate to assign the power consumption of a certain brand of refrigerator one exact value. In this case, HFEs may be a more suitable choice.

Torra [45] defined some operations on HFEs, for any three HFEs h, h_1 and h_2, denoted by

(1) $h^c = \cup_{r\in h}\{1 - r\}$;
(2) $h_1 \cup h_2 = \cup_{r_1\in h_1, r_2\in h_2} \max\{r_1, r_2\}$;
(3) $h_1 \cap h_2 = \cup_{r_1\in h_1, r_2\in h_2} \min\{r_1, r_2\}$.

Based on the relationship between HFEs and AIFVs, Xia and Xu [55] defined the following new operations on HFEs, for any three HFEs h, h_1 and h_2, denoted by

(1) $h^\lambda = \cup_{r\in h}\{r^\lambda\} \quad \lambda > 0$;
(2) $\lambda h = \cup_{r\in h}\{1 - (1 - r)^\lambda\} \quad \lambda > 0$;
(3) $h_1 \oplus h_2 = \cup_{r_1\in h_1, r_2\in h_2}\{r_1 + r_2 - r_1 r_2\}$;
(4) $h_1 \otimes h_2 = \cup_{r_1\in h_1, r_2\in h_2}\{r_1 r_2\}$.