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The decompositions of rank-dependent poverty measures using ordered weighted averaging operators ☆

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ABSTRACT

This paper is concerned with rank-dependent poverty measures and shows that an ordered weighted averaging, hereafter OWA, operator underlies in the definition of these indices. The dual decomposition of an OWA operator into the self-dual core and the anti-self-dual remainder allows us to propose a decomposition for all the rank-dependent poverty measures in terms of incidence, intensity and inequality. In fact, in the poverty field, it is well known that every poverty index should be sensitive to the incidence of poverty, the intensity of poverty and the inequality among the poor individuals. However, the inequality among the poor can be analyzed in terms of either incomes or gaps of the distribution of the poor. And, depending on the side we focus on, contradictory results can be obtained. Nevertheless, the properties inherited by the proposed decompositions from the OWA operators oblige the inequality components to measure equally the inequality of income and inequality of gap overcoming one of the main drawbacks in poverty and inequality measurement. Finally, we provide an empirical illustration showing the appeal of our decompositions for some European Countries in 2005 and 2011.

1. Introduction

Recently, there has been an increasing interest by scholars of applying the Ordered Weighted Averaging, hereafter OWA, operators to several contexts, and in particular, into the economic field. Specifically, the decomposition of the OWA operators as the sum of two parts, proposed by García-Lapresta and Marques Pereira [17], namely the self-dual core and the anti-self-dual remainder, has been applied in a social framework (see García-Lapresta et al. [18] and Aristondo et al. [2], among others). In this paper we show that there exists a connection between a class of poverty measures and OWA operators.

In the evaluation of poverty, two different steps must be taken into account: the *identification* of the poor and the *aggregation* of poverty information in a numerical value. The identification problem has been solved by considering an income threshold, the so-called *poverty line*, which divides a society into poor, whose incomes are below the poverty line, and non-poor, whose incomes are above the poverty line. On the other hand, the aggregation step should essentially be the choice of an appropriate poverty measure. Since the seminal work of Sen [26], a great number of poverty measures have been introduced in the literature, see Clark et al. [11], Chakravarty [7], Foster, Greer and Thorbecke [16], Shorrocks [27], Kakwani [22] among others.

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This paper focuses on the class of *rank-dependent poverty measures* that are those indices defined on poverty gaps for which the weight associated to each individual depends on its position in the distribution.[1] We prove that the normalized version of all the *rank-dependent poverty measures*, such as the *poverty gap ratio*, the two popular indices introduced by Sen [26], the index and the consequent class of indices proposed by Thon [28] and [29], the Kakwani family of indices [21], the Shorrocks index analyzed by Shorrocks [27] and Chakravarty [8] and the S-Gini class introduced by Weymark [30], can be interpreted as OWA operators.

We adopt the methodology proposed by García-Lapresta and Marques Pereira [17] decomposing the OWA operators, underlying in the poverty measures, as the sum of the self-dual core and the anti-self-dual remainder of an OWA operator. In particular, we show that the self-dual core and the anti-self-dual remainder can be reinterpreted as a measure of intensity of poverty and the inequality among the poor, respectively. In fact, Sen [26] mentions in his seminal work that every poverty index should combine three components: the incidence of poverty, the intensity of poverty and the inequality among the poor. In other words, any poverty measure should be a function of the number of poor people in the society, the incidence, the extent of the shortfall of the poor, the intensity, and finally, it should take into account the inequality among the poor. Therefore, we will be able to decompose all the *rank dependent poverty measures* into their three underlying components.

Firstly, we normalize all the *rank-dependent poverty measures* in order to transform them into OWA operators, and then, we decompose the OWA operators into the self-dual core and the anti-self-dual remainder. The incidence component is obtained from the normalization factor, whereas the OWA decomposition gives the remaining two components, namely the intensity and the inequality part.

The archetypical measures for incidence and intensity are the *headcount ratio* and the *income gap ratio*, respectively.[2] However, the inequality among the poor component can also be measured using different inequality measures. In addition, inequality of the poor could refer to the inequality of the income of the poor or to the inequality of the gap of the poor. A crucial requirement in the measurement of inequality is the *Pigou–Dalton principle*. The axiom requires that a transfer of income from a poor individual to a richer one entails an increase in the inequality of the society. This axiom could be interpreted as the counterpart of Sen's [26] *transfer axiom* which demands that a regressive transfer of income has to increase the level of poverty.[3] However, a regressive transfer of income could be also interpreted as a regressive transfer of gap. That is, a transfer of income from a richer to a poorer individuals entails a transfer of gap from the richer on gaps to the poorer on gaps. However, if we focus on shortfalls, the richer on incomes is now the poorer on gaps, and the poorer on incomes is now richer on gaps. Consequently, as we have mentioned, the inequality component obtained decomposing a poverty measure could be defined in terms of incomes or gaps. In this respect, in the literature, different poverty decompositions have been proposed, also for the same poverty index, in terms of incomes or gaps of the poor (see Osberg and Xu [25] and Aristondo et al. [3] among others).

In addition, the choice between income and gap inequality is not innocuous and different choices may lead to contradictory results. That is, the inequality of incomes and the inequality of gaps may have opposite results for the same distribution. However, from the properties of OWA operators inherited by their components, we can conclude that the anti-self-dual remainder, namely the inequality component in our decompositions, is a perfect complementary indicator, i.e., an inequality index that measures equally the inequality of income and the inequality of gap.

A poverty measure that can be decomposed in this way becomes quite important from policy perspective. In fact, policy-makers will be able to analyze the sources of changes in poverty focusing on their three components. In addition, these decompositions allow us to obtain consistent results for incomes and gaps when analyzing the inequality among the poor individuals.

Therefore, we provide an empirical illustration of how our decompositions could be a good instrument for policy-makers so as to allow them to better understand the sources that cause poverty. Using individual level data from the European Union Survey on Income and Living Conditions (EU-SILC), we compute some rank-dependent poverty indices and their decompositions for 25 European countries in two different periods, 2005 and 2011. The analysis will bring out different situations since the increment or reduction on the poverty level for a country could result from different causes. That is, for example, an increase on poverty can be due to an increment on the number of poor, an increase on the intensity of poverty, a growth on the inequality among the poor or some combination of the three. For this reason, we believe that our proposal allows us to identify the source that causes a variation in the poverty index and in consequence to address more efficiency policy aiming at the reduction of poverty.

Our paper is naturally related to two different strands of the literature. Firstly, it is concerned with the literature on aggregation functions and on OWA operators and in particular on their decomposition into the self-dual core and the anti-self-dual remainder. García-Lapresta et al. [18] prove that the normalized version of a class of poverty measures based on the one-parameter family of exponential means can be decomposed using the dual decomposition of an aggregation function into a self-dual core and anti-self-dual remainder. This decomposition lets them to decompose this poverty family into the three poverty components that can be interpreted as incidence, intensity and the inequality among the poor,

[1] The poverty gap of an individual below the poverty line is the difference between the poverty line and its income level. For the ones above the poverty line it is 0.

[2] The *headcount ratio* is the percentage of poor people and the *income gap ratio* is the mean of the relative gap of the poor.

[3] A regressive transfer is a transfer from a poorer to a richer individual when both are poor.