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Forecasting portfolio returns using weighted fuzzy time series methods ☆

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ABSTRACT

We propose using weighted fuzzy time series (FTS) methods to forecast the future performance of returns on portfolios. We model the uncertain parameters of the fuzzy portfolio selection models using a possibilistic interval-valued mean approach, and approximate the uncertain future return on a given portfolio by means of a trapezoidal fuzzy number. Introducing some modifications into the classical models of fuzzy time series, based on weighted operators, enables us to generate trapezoidal numbers as forecasts of the future performance of the portfolio returns. This fuzzy forecast makes it possible to approximate both the expected return and the risk of the investment through the value and ambiguity of a fuzzy number.

We incorporate our proposals into classical fuzzy time series methods and analyze their effectiveness compared with classical weighted fuzzy time series models, using historical returns on assets from the Spanish stock market. When our weighted FTS proposals are used to point-wise forecast portfolio returns the one-step ahead accuracy is improved, also with respect to non-fuzzy forecasting methods.

1. Introduction

The origin of modern portfolio theory is the mean–variance (MV) probabilistic model, where the trade-off between return and risk of the investment is the goal for the decision maker, using probability theory and mathematical optimization as the main tools for achieving an efficient portfolio. Since the introduction of the MV portfolio selection problem [1] by Markowitz in 1952, many different models, approaches, and procedures have been developed in order for suitably managing the uncertain behavior of markets, the experience or beliefs of experts, and the wishes of investors, extending the classical MV model [2–5]. The process of selecting an optimal portfolio may be divided into two stages: the first stage starts with observation of the behavior of returns on assets, incorporating the experience and knowledge of experts, and ends with beliefs about the future performance of available portfolios (through the individual performance of securities and the performance of the portfolio as a whole). The second stage concerns the procedures designed for finding and selecting the portfolio that is optimal with respect to a single objective or multiple objectives, fulfilling the requirements imposed by the investor.

In the field of fuzzy sets, several suitable Soft Computing approaches can be found to the portfolio selection problem in the two stages of the selecting process: in the modeling of beliefs and the incorporation of imprecise knowledge, the approximation of the uncertain performance of future returns on assets and portfolios [6–10], and the development of optimization

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procedures, mainly through fuzzy mathematical programming techniques and evolutionary algorithms [11–15]. In recent years, there has been a growing interest in including realistic constraints (trading and investor preferences) in the portfolio selection models, in incorporating goals alternative and complementary to the standard expected return and variance. With the introduction of these constraints and goals, the portfolio optimization problem becomes a constrained multi-objective decision making problem and new procedures have been designed in order to find efficient portfolios [16–19].

Concerning the first stage of the selecting process, the uncertainty quantification of the future returns in portfolio selection problems, many researchers assume different random distributions of the investment returns (for example, in the MV model, the vector of returns on assets is assumed to be multivariate normally distributed), while others take on different properties of fuzzy logic, but all of them estimate the uncertainty parameters through historical data sets of return on assets. So the ex-ante parameters should be accurately estimated, taking into account their uncertainty during the optimization process. In this paper, we consider the overall performance of portfolios, and to deal with the uncertainty of future returns on a given portfolio we also propose to estimate its ex-post parameters, which will be forecasted using a suitable fuzzy time series method.

Fuzzy modeling of time series has been proposed by Song and Chissom [20–22], introducing time-variant and time-invariant models to forecast the enrollment of students at Alabama University. The major points in their modeling approach are related to the partitioning of the universe of discourse, the establishment of fuzzy relationships from the fuzzy time series (FTS) and the process of forecasting and defuzzification of the outputs. Chen [23] proposed an alternative algorithm, that improved forecasting accuracy and simplified the calculations. His approach has been the basis of almost all FTS methods. For the partitioning of the universe of discourse, different FTS proposals to determine suitable intervals have been considered, given that the determination of effective length of intervals hardly affects forecasting results [24–26]. With respect to specific forecasting proposals most authors follows the scheme originally suggested by Song and Chissom; however, stating appropriate fuzzy relationships is also critical in fuzzy time series. A very interesting approach is proposed in Yu [27], where the weighted fuzzy time series model considers recurrent fuzzy relationships for assigning weights to each individual relationship, outperforming forecasting. Following this line of research, Cheng et al. [28] propose weighting the relationships among the different fuzzy sets (based on frequency), and Lee et al. [29] present other modified weighted versions of Yu's FTS method.

In recent years, fuzzy approaches for solving time series problems have been applied in the financial field for forecasting stock prices and stock market indices, and for modeling business cycles (see, for instance, [27,30–34]). For forecasting portfolio returns, we have previously analyzed the performance of Chen's and Huarng's proposals for partitioning the universe of discourse [23,24], showing the importance of using weighted means in the forecasting process [35]. Recently, Zhou et al. [36] have developed a portfolio optimization model which combines information entropy for measuring the investment risk and fuzzy time series techniques to forecast security returns.

In the present research our main goal is designing a fuzzy time series modeling approach for providing fuzzy forecasts of the future performance of return on a given portfolio, using the historical data set of the portfolio returns. This ex-post information will be useful to the decision analyst once it has been incorporated into the fuzzy mathematical portfolio selection models, which usually approximate the future performance of returns without considering their temporality. However, the forecasts provided by classical fuzzy time series models are usually looking for a single-point value, without measuring the uncertainty on that point-wise forecast. On the other hand, it is well established that for ranking and selecting portfolios we need to know not only their expected return but a measure of the investment risk, usually the variance, the downside risk or the value-at-risk of the investment on a given portfolio.

In previous studies we have dealt with LR-type fuzzy numbers as a useful approximation of the uncertainty of returns on either individual assets or given portfolios, for which possibilistic moments allow us to calculate the expected return, downside risk and some other significant properties [37–39]. A very important type of LR-fuzzy numbers are trapezoidal ones, for which the reference functions are linear. Trapezoidal fuzzy numbers have been intensively used in the fuzzy portfolio selection modeling (see, for instance, [8–10,17] and references therein). Working with trapezoidal fuzzy numbers will allow us to analyze both expected return and risk of a particular investment, which is an appropriate financial approach to the fuzzy portfolio selection problem. Additionally, the aggregation of trapezoidal fuzzy numbers for building the fuzzy forecasts has the nice property that also provides trapezoidal fuzzy numbers, avoiding the drawback of aggregating LR-type fuzzy numbers of different shapes. Therefore, throughout the paper both the linguistic variables in the forecasting process and the fuzzy forecasts are approximated as suitable trapezoidal fuzzy numbers.

The remaining content of this paper is organized as follows. In Section 2, we review the fuzzy portfolio selection problem when possibility theory is applied to measure the uncertainty of the future returns on a portfolio. Section 3 provides a review of fuzzy time series definitions and methods. In Section 4 we include new proposals for improving the fuzzy forecasting of time series. Subsequently, numerical results for a historical data set of the returns on assets from the IBEX35 (the stock index of the Spanish stock market) are provided in Section 5, where we analyze the comparative performance of classical non-fuzzy and fuzzy time series methods with respect to the proposed forecasting approach. It also includes the statistical analysis of the one-step ahead forecasting errors achieved by all those forecasting methods. Conclusions and future research directions are presented in Section 6.