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General fault admittance method solution of a balanced line to line to line to ground fault

J.D. Sakala, J.S.J. Daka *

University of Botswana, Gaborone, Botswana

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ABSTRACT

In the classical approach, balanced line to line to line to ground faults are analysed using knowledge of symmetrical component sequence networks connections. At the fault point negative and zero sequence currents are zero, therefore their networks are not used in finding the sequence currents. The positive sequence voltages at various bus bars and the positive sequence line flows in the networks are calculated from the positive sequence current at the fault point. The phase voltages and voltages are found by transforming the sequence values respectively. The solution proceeds from the knowledge of the sequence networks connections for a particular fault. In contrast, the general fault admittance method solution does not require prior knowledge of the sequence networks connections. Therefore more versatile than the classical methods. The paper presents a procedure for simulating the balanced three phase to ground short circuit, which is a prerequisite for using the general fault admittance method for a balanced line to line to line to ground fault. Based on the general fault admittance method a computer simulation model is developed to analyse a power system with a delta earthed star connected transformer. The results obtained are as accurate as those obtained using the classical approach.

Introduction

The paper presents a method for solving the balanced line to line to line to ground fault using the general fault impedance method. The general fault admittance method differs from the classical approaches based on symmetrical components in that it does not require prior knowledge of how the sequence components of currents and voltages are related [1–6]. In the classical approach, knowledge of how the sequence components are related is required to connect the sequence networks in a prescribed way for a particular fault. Then the sequence currents and voltages at the fault are determined, after which symmetrical component currents and voltages in the rest of the network are calculated. Phase currents and voltages are found by transforming the respective symmetrical component values [7–14].

The fault admittance method is general in the sense that any fault impedances can be represented, provided the special case of a zero impedance fault is catered for.

This paper discusses a procedure for simulating short circuits for the balanced line to line to line to ground fault. Note that this is the most challenging application for the general fault admittance method because of the four zero fault impedances.

Background

A line to line to line to ground fault presents low value impedances, with zero value for direct short circuits or metallic faults, between three phases and between the three phases and ground at the point of a fault in the network. In general, a fault may be represented as shown in Fig. 1.

In Fig. 1, a fault at a busbar is represented by fault admittances in each phase, i.e. the inverse of the fault impedance in the phase, and the admittance in the ground path. Note that the fault admittance for a short-circuited phase is represented by an infinite value, while that for an open-circuited phase is a zero value. In a line to line to line to ground fault the fault is assumed to be between the three phases *a*, *b* and *c* and between them and ground. Thus for a line to line to line to ground fault the admittance Y_{af}, Y_{bf}, Y_{cf} and Y_{gf} are infinite.

A systematic approach for using a fault admittance matrix in the general fault admittance method is given by Sakala and Daka [1–6]. The method is summarized in this paper to give the reader a comprehensive view of the methodology.

* Corresponding author.
E-mail address: sakala@mopipi.ub.bw (J.D. Sakala).

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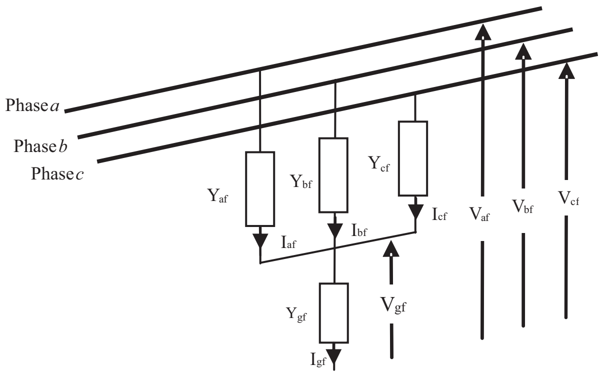

Fig. 1. General fault representation.

The general fault admittance matrix is given by

$$Y_f=\left(\frac{1}{Y_{af}+Y_{bf}+Y_{cf}+Y_{gf}}\right)\times\begin{bmatrix}Y_{af}(Y_{bf}+Y_{cf}+Y_{gf}) & -Y_{af}Y_{bf} & -Y_{af}Y_{cf}\\ -Y_{af}Y_{bf} & Y_{bf}(Y_{af}+Y_{cf}+Y_{gf}) & -Y_{bf}Y_{cf}\\ -Y_{af}Y_{cf} & -Y_{bf}Y_{cf} & Y_{cf}(Y_{af}+Y_{bf}+Y_{gf})\end{bmatrix} \tag{1}$$

Eq. (1) is transformed using the symmetrical component transformation matrix T, and its inverse be T^{-1}, where:

$$T=\begin{bmatrix}1 & 1 & 1\\ \alpha^2 & \alpha & 1\\ \alpha & \alpha^2 & 1\end{bmatrix} \text{ and } T^{-1}=\frac{1}{3}\begin{bmatrix}1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha\\ 1 & 1 & 1\end{bmatrix},$$

in which $\alpha = 1\angle 120°$ is a complex operator.

The symmetrical component fault admittance matrix is given by the product:

$$Y_{fs}=T^{-1}Y_fT$$

The general expression [1–6] for Y_{fs} is given by:

$$Y_{fs}=\frac{1}{Y_{af}+Y_{bf}+Y_{cf}+Y_{gf}}\begin{bmatrix}Y_{fs11} & Y_{fs12} & Y_{fs13}\\ Y_{fs21} & Y_{fs22} & Y_{fs23}\\ Y_{fs31} & Y_{fs32} & Y_{fs33}\end{bmatrix} \tag{2}$$

where

$$Y_{fs11}=Y_{fs22}=\frac{1}{3}Y_{gf}(Y_{af}+Y_{bf}+Y_{cf})+Y_{af}Y_{bf}+Y_{af}Y_{cf}+Y_{bf}Y_{cf}$$

$$Y_{fs33}=\frac{1}{3}Y_{gf}(Y_{af}+Y_{bf}+Y_{cf})$$

$$Y_{fs12}=\frac{1}{3}Y_{gf}(Y_{af}+\alpha^2Y_{bf}+\alpha Y_{cf})-(Y_{bf}Y_{cf}+\alpha Y_{af}Y_{bf}+\alpha^2Y_{af}Y_{cf})$$

$$Y_{fs21}=\frac{1}{3}Y_{gf}(Y_{af}+\alpha Y_{bf}+\alpha^2Y_{cf})-(Y_{bf}Y_{cf}+\alpha^2Y_{af}Y_{bf}+\alpha Y_{af}Y_{cf})$$

$$Y_{fs13}=Y_{fs32}=\frac{1}{3}Y_{gf}(Y_{af}+\alpha Y_{bf}+\alpha^2Y_{cf}) \text{ and}$$

$$Y_{fs31}=Y_{fs23}=\frac{1}{3}Y_{gf}(Y_{af}+\alpha^2Y_{bf}+\alpha Y_{cf})$$

The expressions do simplify considerably depending on the type of fault. For example, consider a balanced three phase fault with $Y_{af} = Y_{bf} = Y_{cf} = Y$.

$$Y_{fs}=\frac{1}{3Y+Y_{gf}}\begin{bmatrix}Y(3Y+Y_{gf}) & 0 & 0\\ 0 & Y(3Y+Y_{gf}) & 0\\ 0 & 0 & YY_{gf}\end{bmatrix}=\begin{bmatrix}Y & 0 & 0\\ 0 & Y & 0\\ 0 & 0 & \frac{YY_{gf}}{3Y+Y_{gf}}\end{bmatrix} \tag{3}$$

There is no coupling between the positive, negative and zero sequence networks. Since there are no negative and zero sequence voltages before the fault there will be no corresponding currents during and after the fault.

Note that in the case that the ground is not involved $Y_{gf} = 0$ and the symmetrical component fault admittance matrix reduces to:

$$Y_{fs}=\begin{bmatrix}Y & 0 & 0\\ 0 & Y & 0\\ 0 & 0 & 0\end{bmatrix} \tag{4}$$

Currents in the fault

At the faulted busbar, say busbar j, the symmetrical component currents in the fault are given by:

$$I_{fsj}=Y_{fs}(U+Z_{sjj}Y_{fs})^{-1}V_{sj}^0 \tag{5}$$

where U is the unit matrix

$$U=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

and Z_{sjj} is the jj^{th} component of the symmetrical component bus impedance matrix

$$Z_{sjj}=\begin{bmatrix}Z_{sjj+} & 0 & 0\\ 0 & Z_{sjj-} & 0\\ 0 & 0 & Z_{sjj0}\end{bmatrix}$$

The diagonal elements $Z_{sjj+}, Z_{sjj-}, Z_{sjj0}$ are the Thevenin's positive, negative and zero sequence impedances, respectively, at the faulted busbar.

Note that the mutual terms are all zero since the network is balanced.

In Eq. (5) V_{sj}^0 is the prefault symmetrical component voltage at busbar j the faulted busbar

$$V_{sj}^0=\begin{bmatrix}V_{sj+}\\ V_{sj-}\\ V_{sj0}\end{bmatrix}=\begin{bmatrix}V_+\\ 0\\ 0\end{bmatrix}$$

where V_+ is the positive sequence voltage before the fault. The negative and zero sequence voltages are zero because the system is balanced prior to the fault.

The phase currents in the fault are then obtained by transformation

$$I_{fpj}=\begin{bmatrix}I_{afj}\\ I_{bfj}\\ I_{cfj}\end{bmatrix}=TI_{fsj} \tag{6}$$

Voltages at the busbars

The symmetrical component voltage at the faulted busbar j is given by

$$V_{fsj}=\begin{bmatrix}V_{j+}\\ V_{j-}\\ V_{j0}\end{bmatrix}=(U+Z_{sjj}Y_{fs})^{-1}V_{sj}^0 \tag{7}$$

The symmetrical component voltage at a busbar i for a fault at busbar j is given by:

$$V_{fsi}=\begin{bmatrix}V_{i+}\\ V_{i-}\\ V_{i0}\end{bmatrix}=V_{si}^0-Z_{sij}Y_{fs}(U+Z_{sjj}Y_{fs})^{-1}V_{sj}^0 \tag{8}$$