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Modified penalty function method for optimal social welfare of electric power supply chain with transmission constraints

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ABSTRACT

Many power system application problems utilize optimal social welfare model as a decision-making tool. In this paper, we tackle the optimal social welfare problem based on a mathematic modeling framework capable to capture the interactions among decision-makers in an electric power supply chain network with consideration of transmission power flows and constraints which are neglected in previous literature, and present a generalized electric power supply chain network equilibrium model. In this model, each player tries to maximize its own profit and competes with others in a noncooperative manner. The Nash equilibrium conditions of these players form a finite-dimensional variational inequality problem (VIP). A modified penalty function (MPF) method is then used to replace conventional extragradient method for solving VIP by transforming VIP into a nonlinear programming problem. Numerical tests are conducted to demonstrate the effectiveness of the MPF for solving the VIP-based social welfare maximization model in the context of an electric power supply chain network.

1. Introduction

The electric power system is a large-scale network of electrical components used to supply, to transmit, and to use electric power. There are many problems arising in the power system area that can be formulated as linear or nonlinear optimization problems. Among these problems, the optimal power flow (OPF) is a complex large-scale nonlinear programming problem that is to find a steady-state operation point which minimizes the cost of power generation or transmission losses, or maximizes social welfare or load capacity, etc. The constraints involved in OPF are power flow equations and operating limitations of components used in the power systems. After restructuring of power networks, several players appear in the electrical market: generator companies, power suppliers, transmission service providers, and consumers (demand markets). Each player has its own operating goals. Most of them try to maximize their own profits and compete with each other in a noncooperative manner, creating complicated decision-making problems. How to model the interactions between these players and how to come up with a satisfactory decision to meet all players' requirements are sophisticated and essential issues to the market operators [1,2]. A mathematic modeling framework capable to capture the interactions among decision-makers in the deregulated power system is of great practical as well as of policy-making importance.

In this paper, an electric power supply chain model with consideration of transmission power flows and constraints is presented to capture the interactions among decision-makers. The Nash equilibrium conditions of these players in this model form a finite-dimensional variational inequality problem (VIP). By solving this VIP, the optimal social welfare is achieved.

In deregulated power markets, the social welfare maximization model has been widely used for solving congestion management problems [3,4], optimal locating and parameter setting of unified power flow controllers (UPFC) or thyristor controlled series compensators (TCSC) [5–7], dynamic security constrained optimal dispatch, security-cost OPF with small-signal stability constraints [8], transmission pricing [9], optimal distributed generation (DG) placement [10], electricity and ancillary services auctions [11], auctions/transactions between generators and large consumers [12], transmission expansion planning [13], evaluating alternative policies regulating wind integration into autonomous energy systems [14], etc. Since the social welfare maximization problem is a branch of OPF problems, most methodologies used for solving it were based on OPF methods. The classical OPF method is based on power flow solution by Newton's method [15] or interior point method (IPM) [3,5,7,8,16]. Newton's method is a gradient adjustment algorithm for obtaining the minimum while using penalty functions to account for dependent constraints [17]. Interior point method utilizes Lagrange method to deal with equalities, barrier function to deal with inequalities, and Newton's method to solve Karush–Kuhn–Tucker (KKT) conditions [16]. In recent years, several studies

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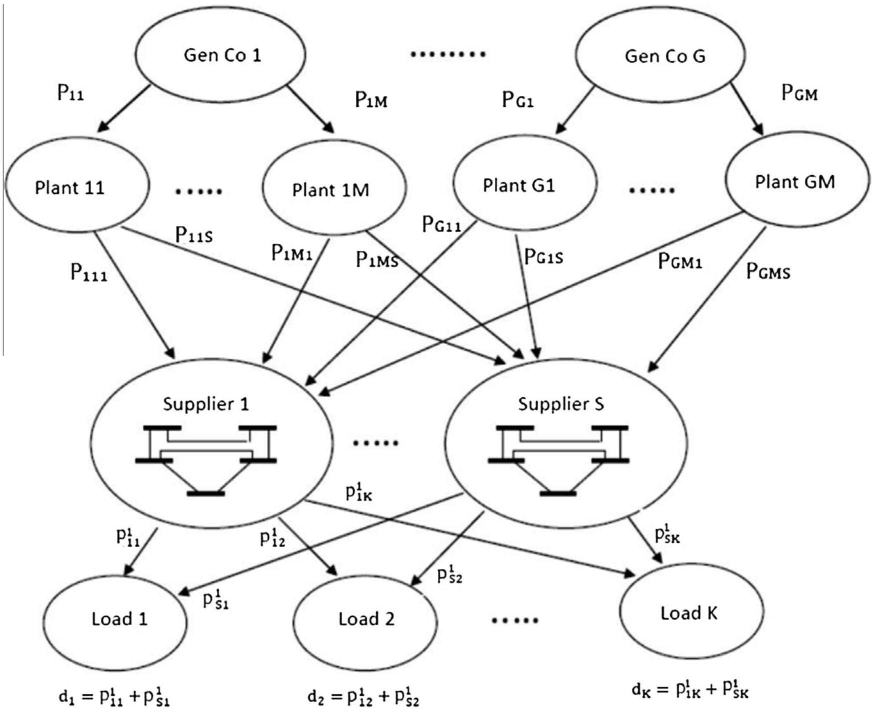

Fig. 1. Electric power supply chain with transmission networks.

tried to solve OPF using non-conventional approaches. Barbosa proposed a modified artificial immune system (AIS) algorithm to find the multiple optima in an OPF problem [18]. Bose presented tree networks to solve the convex dual problem [19]. Sivasubramani developed a sequential quadratic programming (SQP) and differential evolution (DE) algorithm to solve OPF more effectively than the classical evolutionary algorithms [20]. Nabavi [6] used a genetic algorithm for finding the optimal location and size of TCSC for congestion management with aim of increasing social welfare while cost of TCSC was incorporated. Particle swarm optimization techniques were used by Divshali et al. to solve dynamic security constrained optimal dispatch in deregulated power systems [21], and by Fard et al. to solve transmission congestion problem based on the social welfare maximization model in restructured electricity market [4]. Nagurney and Liu [22] tackled OPF based on the electric power supply chain network model and demonstrated that electric power supply chain network equilibrium problems can be reformulated and solved as transportation network equilibrium problems. They proved that the variational inequality problem (VIP) formulations of the governing equilibrium conditions coincide with the corresponding variational inequalities of transportation network equilibrium problems over appropriately constructed supernetworks. Along the path of electric supply chain model, Woolley et al. [23] developed a multipollutant permit trading model in which different technologies associated with electric power production were presented. However, these based on electric supply chain models neglected the power flows of transmission lines and transmission constraints. To improve this modelling shortcoming, we propose in this paper an extended electric power supply chain model with transmission power flows and transmission constraints. Moreover, instead of using the conventional time-consuming extragradient method [24] for VIP, we tackled the problem based on a modified penalty function (MPF) method [25]. The classical penalty method replaces a constrained optimization problem by a series of unconstrained problems with large penalty parameters. The larger the penalty parameter is, the worse the condition number of the Hessian matrix the near optimum will be, causing numerical difficulty for convergence. Deb and Datta combined a bi-objective evolutionary approach with the penalty function methodology in a manner complementary to each other [26]. The bi-objective optimization approach provides a good estimate of the penalty parameter, while the unconstrained penalty function approach using classical means provides the overall hybrid algorithm its convergence property. Bester and Hansen presented a penalized objective function that reduces the bias in the resulting point estimates [27] effectively for an empirical study of insider trading activity. Liu et al. [25] presented a modified penalty function (MPF) method to minimize a nonlinear programming subject to inequality constraints. This method is a combination of the penalty method and the Lagrangian method. This MPF was used for solving VIP in this paper because of its good properties: the smoothness of the initial functions and positive definite second-order Hessian matrix.

This paper is organized as follows. The optimal social welfare problem of an electric power supply chain with transmission constraints is introduced in Section 2. In Section 3, the MPF method for solving the presented problem is presented. The numerical results using the MPF and QP-based methods are reported in Section 4. Conclusions are drawn in the end.

2. Optimal social welfare of electric power supply chain with transmission constraints

A modeling and computational framework is presented in this paper for the determination of optimal social welfare in the context of electric power supply chain with consideration of transmission power flows and constraints which are neglected in [22,23,28]. In order to achieve this goal, a modified electric power supply chain network equilibrium model is used. Under deregulation, there are several players in electrical market: generator companies (Gen Cos), power suppliers, transmission service providers, and consumers (demand markets). Each player in this model tries to maximize its own profit and competes with others in a noncooperative manner.