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Unit commitment problem: A new formulation and solution method

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ABSTRACT

In this paper, we present a new formulation and solution method for the well known *unit commitment problem* (UCP) for scheduling the thermal generators in a day-ahead electricity market. Compared to the traditional approach, our approach has several advantages such as: (a) reducing the combinatorial complexity (i.e., the size of the binary state space) significantly, (b) eliminating the need for linearizing the constraints associated with the minimum ON time and minimum OFF time for any thermal generator, (c) eliminating the need for defining new binary decision variables to represent the startup and shutdown decisions for any thermal generator in each hour and (d) eliminating the need to linearize the non-linear cost functions associated with any thermal generator (e.g., time dependent exponential startup cost function). According to our formulation, the UCP can be stated as finding a feasible path of ON–OFF states for each generator (i.e., a sequence of unit commitment states that satisfy the corresponding minimum ON time and minimum OFF time constraints over the scheduling horizon) such that the total generation cost is minimized while meeting the demand and reserve requirement in each hour for the next day. We show how a near optimal solution for the UCP can be constructed using our solution method which is based on the *Lagrangian relaxation* (LR) method. Although only a near optimal solution is found, we show that our solution is comparable to that obtained when the UCP is modeled as a *mixed integer linear program* (MILP).

1. Introduction

OVER the past two decades there has been a great deal of research in power generation and planning. One of the fundamental issues still remains finding the minimum cost schedule of the thermal generators (e.g., coal/nuclear power plants) which form the bulk of today's power supply. The thermal generators require elaborate planning and scheduling on a day-to-day basis apart from regular monitoring and recourse actions on shorter time scales (e.g., power levels are adjusted on hourly basis) [1]. The scheduling operation for each day is usually performed on a day-ahead basis using information like forecasted demand for each hour, desired reserve power for each hour (i.e., backup power to balance demand changes in real-time), generator specific characteristics (e.g., ramping limits, minimum ON time and minimum OFF time), initial ON/OFF statuses and initial power levels corresponding to each generator in the system. This scheduling operation is popularly referred to as the *unit commitment problem* (UCP) [2,3]. The unit commitment problem is considered as an NP hard non-convex optimization problem owing to the presence of binary decision variables which increase with the size of the system. The combinatorial complexity[1] increases with an increase in the number of independent binary decision variables which are often introduced in the existing works to linearize the objective function and the constraints of the UCP. Most works have used binary decision variables to only represent the unit commitment status (i.e., ON/OFF state) of a generator in each hour [3–9]. Besides using binary decision variables to represent the unit commitment status of every generator in each hour, several works also introduced separate binary decision variables to represent the startup and shutdown decisions for every generator in each hour [10–13] (i.e., three binary decision variables were used to represent a generator's state in each hour). Binary decision variables were also introduced when representing the nonlinear cost functions of a generator (e.g., approximating an exponential time dependent startup cost function [12]) in a linear form which is acceptable to a *mixed integer linear programming* (MILP) solver. When the state space of the binary decision variables becomes very large, it affects the performance of an MILP solver in terms of memory and computation time [6]. Tight coupling constraints among different types of binary decision variables can reduce the combinatorial complexity to a certain extent. For example, in [12], the three types of binary decision variables associated with every generator (i.e., those representing the startup state, shutdown state and the unit commitment status in each hour) were coupled by linear constraints such that the binary state space reduced to that corresponding to binary decision variables representing the unit commitment status in different hours of the scheduling horizon. However, it should be noted that even with one binary decision variable representing a generator's state in each hour of the

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[1] Combinatorial complexity means the number of combinations of the binary decision variables taken over the scheduling horizon or the size of the binary state space.

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scheduling horizon, the combinatorial complexity can still be very high.

Hence, in this paper, we propose a new formulation and solution method for the unit commitment problem to significantly reduce the combinatorial complexity. The key contributions of our paper are given below:

- We significantly reduce the combinatorial complexity of the UCP by exploiting the structure of the proposed UCP formulation and our solution method (explained later). Our solution method efficiently solves the proposed UCP in reasonable computation time.
- We do not require linearizing the minimum ON time and minimum OFF time constraints corresponding to any generator.
- We do not need new binary decision variables to represent the startup and shutdown decisions for every generator in each hour. Therefore, we do not require tight coupling constraints such as those defined in [12].
- Unlike the existing works which are based on MILP formulation, we do not linearize the non-linear cost functions associated with every generator (e.g., the time dependent exponential start up cost function). Thus, the accuracy of the cost functions are preserved.

Unlike the existing works, we do not treat the unit commitment status of a generator in any given hour as a binary decision variable (i.e., 1 (0) denotes ON (OFF) state of a generator in a given hour). Instead, we focus on the various sequences of ON–OFF states that are available to a generator over the scheduling horizon of 24 h. For any generator, we define a *feasible path* as a sequence of ON–OFF states which satisfy the minimum ON time and minimum OFF time constraints corresponding to that generator. Each feasible path is divided into two feasible sub-paths where the first feasible sub-path contains the unit commitment statuses (i.e., ON/OFF states) for the first 12 h while the second feasible sub-path contains the unit commitment statuses for the remaining 12 h interval.[2] Each feasible sub-path is associated with one binary decision variable which indicates the selection of the corresponding feasible sub-path (i.e., when the binary decision variable corresponding to a feasible sub-path is set equal to 1 it means that that feasible sub-path is chosen). Thus, when a generator selects a feasible path across the scheduling horizon, it has to set the corresponding binary decision variables for both the 12-h intervals equal to one. The objective of our UCP formulation is to find one feasible path per generator[3] such that it leads to the minimization of the total generation side costs (i.e., sum of the production cost and startup cost for each generator over the scheduling horizon) while meeting the demand and reserve requirement in each hour. Section 4.1 gives the detailed procedure for: (a) precalculating the feasible sub-paths for each generator, (b) associating the sub-paths in the first interval to the sub-paths in the second interval, (c) how the unit commitment statuses are implicitly represented by the binary decision variable associated with each sub-path and (d) precalculating and representing the total startup cost corresponding to any feasible path for a generator. It is worth mentioning that the precalculations described in Section 4.1 are carried out offline and only once.

The rest of the paper is organized as follows. In Section 2, we present a brief overview of some of the UCP formulations from the literature. In Section 3, we outline the traditional UCP formulation (most existing works use a linearized version of the traditional UCP formulation). In Section 4, we present the proposed UCP formulation and solution method for finding a near optimal primal solution. We will show the advantage of our approach over the traditional UCP formulation using a simple illustration. For simplicity of presentation, the transmission flow constraints [14,15] are not included although their inclusion does not affect the complexity of our solution method. In Section 5, we provide some numerical results and discussion. A comparison between our approach and an MILP approach [7] is presented. Finally, we end with some concluding remarks in Section 6.

2. Related work

The literature on UCP is vast and there are many ways of classifying an UCP formulation (e.g., a classification based on different solution methods can be found in [16]). It is worth noting that several versions of the UCP can be found in the literature such as scheduling thermal generators based on emission costs [17] and prohibited operating zones [18], hydro-thermal scheduling [19,20], and thermal/hydro scheduling in the presence of wind power [21,22]. However, we only consider day-ahead scheduling of thermal generators (which we refer to as the *traditional UCP*). In the future, we will extend our formulation to other versions of the UCP.[4] Here, we briefly discuss a few existing works based on whether objective function was linear [7,8,11–13] or nonlinear (i.e., cost functions are not approximated) [4,5,23,24]. Mostly the linear form UCP was solved using MILP solvers (which use branch and bound/cut strategy for finding the solution) while most nonlinear form UCP were solved using a combination of the LR method and dynamic programming (DP) method.

An MILP formulation was presented in [7]. The production cost function and startup cost functions in the objective were linearized. Piecewise linear segments were used for approximating the quadratic production cost function for any generator whereas the time dependent startup cost function for any generator was represented by a non-decreasing step function. The binary decision variables in the formulation represented the unit commitment status of a generator in each hour. The minimum ON time and minimum OFF time constraints which depend on the total number of hours a generator was online and offline respectively were linearized using different binary decision variables.

Similar to [7], an MILP formulation was proposed in [8]. However, the linear form for the minimum ON time and minimum OFF time constraints differed from that in [7]. Another difference between [7,8] was that in [8] the startup cost function was assumed to be a constant for each generator (i.e., startup cost did not depend on the total number of hours a generator was offline).

The UCP was presented as an MILP formulation in [11] where the quadratic production cost function was approximated by piecewise linear segments similar to [7]. However, the startup cost function was modeled as a time dependent linear function. For an offline generator, the startup cost switched from a lower constant value (equal to the hot start cost) to a higher constant value (equal to the hot start cost plus cold start cost) as the total offline duration exceeded some threshold time specified for that generator. The

[2] Further dividing the scheduling horizon into smaller intervals can result in fewer feasible sub-paths in each interval; however, some of the feasible sub-paths can be lost (e.g., minimum ON time and minimum OFF time is 10 h but scheduling horizon is divided into more than 2 intervals). We use two 12-h intervals because most generators have $T^{i,on}$ and $T^{i,off}$ less than 12 h.

[3] Unit commitment decisions for generators which do not have time dependent constraints can be made independently on an hourly basis. Hence they do not need feasible path declaration.

[4] Stochastic formulation can be composed to account for uncertainty using scenarios and solving a deterministic formulation corresponding to each scenario (note that the precalculations given in Section 4.1 will be scenario specific). If the uncertainty is due to forecast errors in wind power and load, then scenarios can be constructed based on the approach in [22]. If the uncertainty is due to a generator's outage, then scenarios can be constructed based on the approach in [21].