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# Deriving the priority weights from incomplete hesitant fuzzy preference relations in group decision making

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ABSTRACT

The concept of hesitant fuzzy preference relation (HFPR) has been recently introduced to allow the decision makers (DMs) to provide several possible preference values over two alternatives. This paper introduces a new type of fuzzy preference structure, called incomplete HFPRs, to describe hesitant and incomplete evaluation information in the group decision making (GDM) process. Furthermore, we define the concept of multiplicative consistency incomplete HFPR and additive consistency incomplete HFPR, and then propose two goal programming models to derive the priority weights from an incomplete HFPR based on multiplicative consistency and additive consistency respectively. These two goal programming models are also extended to obtain the collective priority vector of several incomplete HFPRs. Finally, a numerical example and a practical application in strategy initiatives are provided to illustrate the validity and applicability of the proposed models.



## 1. Introduction

Since the introduction of fuzzy sets by Zadeh [45], several extensions and generalizations have been proposed (see Ref. [6]), including the intuitionistic fuzzy sets [5], interval-valued fuzzy sets [44], type-2 fuzzy sets [24], type *n* fuzzy sets [15] and fuzzy multisets [23]. Another extension of fuzzy sets is called hesitant fuzzy sets (HFSs), which were firstly introduced by Torra [31]. The motivation for introducing HFSs is that it is sometimes difficult to determine the membership of an element into a set, and in some circumstances, this difficulty is because there is a set of possible values.

HFSs are a new effective tool used to express human's hesitancy in daily life and have been receiving an increasing amount of attention in different areas, mainly in group decision making (GDM) [7,12,27,29,34,43,46]. Xia and Xu [33] defined the hesitant fuzzy preference relations (HFPRs) and hesitant multiplicative preference relations (HMPRs), which are based on the fuzzy preference relations and multiplicative preference relations, respectively. There are two more types of preference relations: interval-valued hesitant preference relations (IVHPRs) [7] and hesitant fuzzy linguistic preference relations (HFLPRs) [48] which are based on the hesitant fuzzy linguistic term sets [27,28]. Relationships of HFSs with other types of fuzzy sets can be found in [26] (see Section 5) and a historical overview of the fuzzy sets extensions analyzing their relationship can be found in [6].

The key motivating factors to introducing the concept of incomplete HFPR can be summarized as follows: (1) all of the aforementioned preference relations (HFPR, IVHPR, HMPR and HFLPR) do not consider the incomplete information. (2) In many real decision making problems, due to time pressure, lack of knowledge, and the DM's limited expertise related with the problem domain [1–4,8,10,17–19,37,40], the DMs may obtain a preference relation with incomplete entries. Incomplete HFPR do not merely permit the DMs to provide all of the possible values, but also allow them to give null values when comparing two alternatives. (3) It can enrich the theoretical system of preference relations. Zhang et al. [47] proposed two estimation procedures to estimate the missing information in an expert's incomplete HFPR, which are based on Xu et al.'s [40] models.

GDM problems consist in finding the best alternative(s) from a set of feasible ones according to the preference relations provided by a group of experts. In order to rank the alternatives, one direct method is to derive priorities from the group preference relations. Dong et al. [14] developed a framework to deal with the individual selection problem of the numerical scale and prioritization method in AHP. Dong and Herrera-Viedma [13] proposed a

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consistency-driven automatic methodology to set interval numerical scales of 2-tuple linguistic term sets in the decision making problems.

Up to now, there has been no investigation of deriving the priority weights from the incomplete HFPR. The aim of this paper is to propose some models to obtain priorities from incomplete HFPRs which are based on multiplicative consistency [9,11,30] and additive consistency [3,8,38] of fuzzy preference relations [19,30,38,41], respectively. As the DM gives a HFPR, each comparison has several values and the DM is hesitant on these values, we should abstract the most reasonable information from these values. That is we could derive the most consistent fuzzy preference relation from the HFPR to make decision. This is the main idea of the paper, and it is a new idea to deal with HFPR.

These models are programming models for multiplicative consistency incomplete HFPR and additive consistency incomplete HFPR respectively. Furthermore, we extend these programing models to obtain the collective priority vector of several incomplete HFPRs for the sake of application in GDM process. To show the potential of this proposal, we introduce two illustrative cases of study to show the effectiveness of the developed models.

The remained of this paper is organized as follows. Section 2 briefly reviews some basic knowledge on fuzzy preference relation, HFS and HFPR. Section 3 introduces the concepts of incomplete HFPR, acceptable incomplete HFPR, multiplicative consistent incomplete HFPR and additive consistency incomplete HFPR. In Section 4, we develop some new goal programming models to derive the priority weights from multiplicative consistency incomplete HFPR and additive consistency incomplete HFPR. Section 5 provides a numerical example and a case study in GDM concerning strategy initiatives showing validity and applicability of the proposed models. Some conclusions are pointed out in Section 6.

## 2. Preliminaries

In this section, we will give the definitions of fuzzy preference relation, hesitant fuzzy set, hesitant fuzzy element and hesitant fuzzy preference relation.

Denote $N=\{1,2,\ldots,n\}$, $M=\{1,2,\ldots,m\}$. Let $X=\{x_1,x_2,\ldots,x_n\}$ $(n\geq 2)$ be a finite set of alternatives, where $x_i$ denotes the $i$th alternative.

### 2.1. Fuzzy preference relation

**Definition 1** [20]**.** Let $R=(r_{ij})_{n\times n}$ be a preference relation, then $R$ is called a fuzzy preference relation, if

$$r_{ij}\in[0,1],\quad r_{ij}+r_{ji}=1,\quad r_{ii}=0.5 \text{ for all } i,j\in N. \tag{1}$$

**Definition 2** [30]**.** Let $R=(r_{ij})_{n\times n}$ be a fuzzy preference relation, then $R$ is called a multiplicative consistency fuzzy preference relation, if the following multiplicative transitivity is satisfied:

$$r_{ik}r_{kj}r_{ji}=r_{ki}r_{jk}r_{ij} \quad \textit{for all } i,j,k\in N. \tag{2}$$

**Definition 3** [9,30]**.** If $R=(r_{ij})_{n\times n}$ is a multiplicative consistency fuzzy preference relation, then such a preference relation is given by

$$r_{ij}=\frac{w_i}{w_i+w_j},\quad i,j\in N. \tag{3}$$

where $W=(w_1,w_2,\ldots,w_n)^T$ is the priority weighting vector for the fuzzy preference relation $R=(r_{ij})_{n\times n}$ and $\sum_{i=1}^n w_i=1$, $w_i=0$, $i\in N$.

**Definition 4** [30]**.** Let $R=(r_{ij})_{n\times n}$ be a fuzzy preference relation, then $R$ is called an additive consistency fuzzy preference relation, if the following additive transitivity is satisfied:

$$r_{ij}=r_{ik}-r_{jk}+0.5 \text{ for all } i,j,k\in N. \tag{4}$$

For the additive consistency fuzzy preference relation, there is a function between the element $r_{ij}$ and the weights $w_i$ and $w_j$. The function is obtained as follows.

**Lemma 1** [39]**.** *Let $R=(r_{ij})_{n\times n}$ be a fuzzy additive transitive preference relation, $W=(w_1,w_2,\ldots,w_n)^T$ be the corresponding weighting vector, where $0\leq w_i\leq 1$, then there exists a positive number $\beta$, and such a relation can be expressed as follows*:

$$r_{ij}=0.5+\beta(w_i-w_j). \tag{5}$$

**Remark 1.** Lemma 1 denotes that there is an explicit function relation between $r_{ij}$ and the ranking values $w_i$ and $w_j$. Chiclana et al. [11] constructed a similar relationship between the additive reciprocal preference relation and utility values. Tanino [30] first established the above correspondence where $\beta$ always equals to 0.5, but it was later shown that the correspondence is not always valid from different perspectives [16,21,22,35,36,39]. In the following, we will determine the value of $\beta$.

**Theorem 1.** *If the priority vector of the additive transitive perfectly consistency fuzzy preference relation $R$ is derived by normalizing rank aggregation method, then $\beta=\frac{n-1}{2}$.*

**Proof.** If the priority vector of the additive transitive perfectly consistency fuzzy preference relation $R$ is derived by normalizing rank aggregation method [35], then

$$w_i=\frac{\sum_{k=1}^n r_{ik}-0.5}{\sum_{i=1}^n\sum_{k=1,k\neq i}^n r_{ik}}=\frac{\sum_{k=1}^n r_{ik}-0.5}{\frac{n(n-1)}{2}},\quad i\in N. \tag{6}$$

$$w_j=\frac{\sum_{k=1}^n r_{jk}-0.5}{\sum_{i=1}^n\sum_{k=1,k\neq j}^n r_{jk}}=\frac{\sum_{k=1}^n r_{jk}-0.5}{\frac{n(n-1)}{2}},\quad i\in N. \tag{7}$$

Introducing Eqs. (6) and (7) into Eq. (5), then

$$r_{ij}=\beta(w_i-w_j)+0.5$$
$$=\beta\frac{\sum_{k=1}^n(r_{ik}-r_{jk})}{\frac{n(n-1)}{2}}+0.5$$

Since

$$r_{ij}=r_{ik}-r_{jk}+0.5$$

then

$$r_{ij}=\beta\frac{\sum_{k=1}^n(r_{ij}-0.5)}{\frac{n(n-1)}{2}}+0.5=\beta\frac{nr_{ij}-n/2}{\frac{n(n-1)}{2}}+0.5 \tag{8}$$

So we can get $\beta=\frac{n-1}{2}$, which complete the proof. □

That is to say, the relationship between $r_{ij}$ and $w_i-w_j$ is:

$$r_{ij}=0.5+\frac{n-1}{2}(w_i-w_j). \tag{9}$$

**Remark 2.** In addition, due to the fact that $0<r_{ij}<1$, we have $0<0.5+\frac{n-2}{2}(w_i-w_j)<1$, that is $-1/(n-1)<w_i-w_j<1/(n-1)$.

### 2.2. Hesitant fuzzy set

Torra [31] originally developed the definition of hesitant fuzzy sets (HFSs) as follows.

**Definition 5.** [31,32]. Let $X$ be a reference set, an HFS on $X$ is defined in terms of a function $h_A(x)$ that returns a non-empty subset of [0,1] when it is applied to $X$, i.e.

$$A=\{\langle x,h_A(x)\rangle | x\in X\}. \tag{10}$$

where $h_A(x)$ is a set of some different values in [0,1], representing the possible membership degrees of the element $x\in X$ to $A$. $h_A(x)$ is called a hesitant fuzzy element (HFE), a basic unit of HFS.