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Note on "Hesitant fuzzy prioritized operators and their application to multiple attribute decision making"

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ABSTRACT

Motivated by the idea of prioritized aggregation (PA) operators, Wei (2012) developed two hesitant fuzzy prioritized aggregation (HFPA) operators, and discussed their desirable properties, but the definitions for the HFPA operators and their properties still need to be improved. In this short note, a numerical example is given to show that the idempotency of the HFPA operators suffers from certain shortcomings. Then, based on some adjusted operations on the hesitant fuzzy elements (HFEs), two improved aggregation operators are investigated to aggregate the collective of attribute values. We further prove that the improved operators have the properties of idempotency and boundedness. Finally, the comparison with the method proposed by Wei (2012) is performed to demonstrate that the proposed information aggregation method is both valid and practical to deal with decision making problems.

1. Introduction

Since Zadeh introduced the fuzzy sets (FSs) [2], fuzzy sets have been achieved a great success in various fields. The concept of intuitionistic fuzzy sets (IFSs) [3] put forward by Atanassov is a generalization of the fuzzy set. Atanassov and Gargov further introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) [4], the components of which are intervals rather than exact numbers. Torra [5] proposed the concept of hesitant fuzzy set (HFS) considered as another generalization of FSs, which permits the membership degree having a collection of possible values. Wei [1] extended the prioritized averaging (PA) operator to accommodate the situations where the input arguments are hesitant fuzzy information, and developed two prioritized aggregation operators. He further studied some desirable properties of the proposed operators. However, a close examination demonstrates that some properties suffer from serious drawbacks. The purpose of this note is to point out and correct errors in the properties of HFPA operators.

2. Preliminaries

By the relationship between the HFEs and intuitionistic fuzzy values (IFVs), Xia and Xu [6] defined some operations on the HFEs.

Combined with the PA operator, Wei [1] developed two HFPA operators, including the hesitant fuzzy prioritized weighted average (HFPWA) operator and the hesitant fuzzy prioritized weighted geometric (HFPWG) operator.

Definition 1. (See Wei [1], Definitions 7 and 8) Let $h_j, j = 1, 2, \dots, n$ be a collection of HFEs, then the HFPWA operator and the HFPWG operator are defined as follows, respectively:

$$HFPWA(h_1, h_2, \dots, h_n) = \mathop{\oplus}_{j=1}^{n} \left(\frac{T_j}{\sum_{j=1}^{n} T_j} h_j \right)$$
$$= \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \dots, \gamma_n \in h_n} \left\{ 1 - \prod_{j=1}^{n} (1-\gamma_j)^{\frac{T_j}{\sum_{j=1}^{n} T_j}} \right\}, \quad (1)$$

$$HFPWG(h_1, h_2, \dots, h_n) = \mathop{\otimes}_{j=1}^{n} h_j^{\frac{T_j}{\sum_{j=1}^{n} T_j}}$$
$$= \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \dots, \gamma_n \in h_n} \left\{ \prod_{j=1}^{n} (\gamma_j)^{\frac{T_j}{\sum_{j=1}^{n} T_j}} \right\}, \quad (2)$$

where $T_1 = 1, T_j = \prod_{k=1}^{j-1} s(h_k), j = 2, \dots, n,$ and $s(h_k)$ is the score values of $h_k, k = 1, 2, \dots, n$.

Then, Wei [1] proved that both the HFPWA operator and the HFPWG operator are idempotent.

Theorem 1. (Idempotency, see Wei [1], Theorems 2 and 6) Let $h_j, j = 1, 2, \dots, n,$ be a collection of HFEs, where $T_1 = 1,$

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$T_j = \prod_{k=1}^{j-1} s(h_k)$, $j = 2, \ldots, n$, and $s(h_k)$ is the score values of h_k, $k = 1, 2, \ldots, n$. If all h_j, $j = 1, 2, \ldots, n$, are equal, i.e. $h_j = h$ for all j, then

$$HFPWA(h_1, h_2, \ldots, h_n) = h, \tag{3}$$

$$HFPWG(h_1, h_2, \ldots, h_n) = h. \tag{4}$$

3. Numerical example

Now, we furnish the following example to demonstrate that Theorem 1 is technically incorrect.

Example 1. Suppose that h_1 and h_2 are two HFEs, and $h_1 = h_2 = h = \{0.35, 0.50, 0.77\}$, then the score values of h_1 and h_2 are $s(h_1) = s(h_2) = 0.54$, and then

$$T_1 = 1, T_2 = \prod_{k=1}^{2-1} s(h_k) = s(h_1) = 0.54.$$

According to Definition 1, it follows that

$$HFPWA(h_1, h_2) = HFPA(h, h) = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ 1 - \prod_{j=1}^{2} (1 - \gamma_j)^{\frac{T_j}{\sum_{j=1}^{2} T_j}} \right\}$$
$$= \Big\{ 1 - (1 - 0.35)^{\frac{1}{1+0.54}} \times (1 - 0.35)^{\frac{0.54}{1+0.54}},$$
$$1 - (1 - 0.35)^{\frac{1}{1+0.54}} \times (1 - 0.50)^{\frac{0.54}{1+0.54}},$$
$$1 - (1 - 0.35)^{\frac{1}{1+0.54}} \times (1 - 0.77)^{\frac{0.54}{1+0.54}},$$
$$1 - (1 - 0.50)^{\frac{1}{1+0.54}} \times (1 - 0.35)^{\frac{0.54}{1+0.54}},$$
$$1 - (1 - 0.50)^{\frac{1}{1+0.54}} \times (1 - 0.50)^{\frac{0.54}{1+0.54}},$$
$$1 - (1 - 0.50)^{\frac{1}{1+0.54}} \times (1 - 0.77)^{\frac{0.54}{1+0.54}},$$
$$1 - (1 - 0.77)^{\frac{1}{1+0.54}} \times (1 - 0.35)^{\frac{0.54}{1+0.54}},$$
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$$1 - (1 - 0.77)^{\frac{1}{1+0.54}} \times (1 - 0.77)^{\frac{0.54}{1+0.54}} \Big\}$$
$$= \{0.3500, 0.4071, 0.5486, 0.4518, 0.5000, 0.6192, 0.6689, 0.6980, 0.7700\}.$$

Therefore, the score value of $HFPWA(h_1, h_2)$ can be calculated as

$$s(HFPWA(h_1, h_2)) = 0.5571,$$

then

$$s(HFPWA(h_1, h_2)) = 0.5571 > 0.54 = s(h),$$

which indicates that

$$HFPWA(h_1, h_2) > h. \tag{5}$$

On the other hand, by the HFPWG operator in Definition 1, it is obtained that

$$HFPWG(h_1, h_2) = HFPG(h, h) = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \prod_{j=1}^{2} (\gamma_j)^{\frac{T_j}{\sum_{j=1}^{2} T_j}} \right\}$$
$$= \Big\{ 0.35^{\frac{1}{1+0.54}} \times 0.35^{\frac{0.54}{1+0.54}}, 0.35^{\frac{1}{1+0.54}} \times 0.50^{\frac{0.54}{1+0.54}}, 0.35^{\frac{1}{1+0.54}} \times 0.77^{\frac{0.54}{1+0.54}},$$
$$0.50^{\frac{1}{1+0.54}} \times 0.35^{\frac{0.54}{1+0.54}}, 0.50^{\frac{1}{1+0.54}} \times 0.50^{\frac{0.54}{1+0.54}}, 0.50^{\frac{1}{1+0.54}} \times 0.77^{\frac{0.54}{1+0.54}},$$
$$0.77^{\frac{1}{1+0.54}} \times 0.35^{\frac{0.54}{1+0.54}}, 0.77^{\frac{1}{1+0.54}} \times 0.50^{\frac{0.54}{1+0.54}}, 0.77^{\frac{1}{1+0.54}} \times 0.77^{\frac{0.54}{1+0.54}} \Big\}$$
$$= \{0.3500, 0.3966, 0.4615, 0.4412, 0.5000, 0.5817, 0.5840, 0.6618, 0.7700\}.$$

Thus, we calculate the score value of $HFPWG(h_1, h_2)$ and get that

$$s(HFPWG(h_1, h_2)) = 0.5274,$$

and then

$$s(HFPWG(h_1, h_2)) = 0.5274 < 0.54 = s(h),$$

which indicates that

$$HFPWG(h_1, h_2) < h. \tag{6}$$

Example 1 demonstrates that Theorem 1 of the HFPA operators cannot be tenable, which suffer from serious drawbacks. In this case, the operations on the HFEs need to be improved. In the following section, some adjusted operations for HFEs are presented, and two new HFPA operators are developed, which satisfy the properties of idempotency and boundedness.

4. The improved operators and their properties

In this section, some adjusted operations on the HFEs are reviewed, and two new improved aggregation operators based on these adjusted operations are investigated to aggregate the collective of attribute values. We further prove that the improved operators have the properties of idempotency and boundedness. In the end, an example shows that our method is easier than that of Wei [1] in some cases.

Remark 1. Notice that the number of values in different HFEs may be different. Suppose that l_h stands for the number of values in h, then the following assumptions are made:

(R1) All the elements in each HFE h are arranged in decreasing order, and $\gamma^{(i)}$ is the ith largest value in h;
(R2) If $l_h \neq l_g$, then $l = \max\{l_h, l_g\}$. To have a correct comparison, the two HFEs h and g should have the same length. If there are fewer elements in h than in g, an extension of h should be considered optimistically by repeating its maximum element until it has the same length with g;
(R3) For convenience, we assume that all the HFEs have the same length l, i.e., $h = \{\gamma^{(1)}, \gamma^{(2)}, \ldots, \gamma^{(l)}\}$.

Now, let's review some adjusted operations on the HFEs as follows:

Definition 2. **[7]** Suppose that h_1, h_2 and h are three HFEs, then

(1) $h_1 \dot{\oplus} h_2 = \bigcup_{i=1}^{l} \{\gamma_1^{(i)} + \gamma_2^{(i)} - \gamma_1^{(i)}\gamma_2^{(i)}\}$;
(2) $h_1 \dot{\otimes} h_2 = \bigcup_{i=1}^{l} \{\gamma_1^{(i)}\gamma_2^{(i)}\}$;
(3) $\lambda h = \bigcup_{i=1}^{l} \{1 - (1 - \gamma^{(i)})^{\lambda}\}, \lambda > 0$;
(4) $h^{\lambda} = \bigcup_{i=1}^{l} \{(\gamma^{(i)})^{\lambda}\}, \lambda > 0$.

According to the Definition 2, we know that h_1, h_2, $h_1 \dot{\oplus} h_2$ and $h_1 \dot{\otimes} h_2$ have the same length l.

Example 2. Suppose that $l = 3$, $\lambda = 2$, h_1 and h_2 are two HFEs, and $h_1 = \{0.77, 0.50, 0.35\}$, $h_2 = \{0.86, 0.59, 0.22\}$, then we have

(1) $h_1 \dot{\oplus} h_2 = \bigcup_{i=1}^{3} \{\gamma_1^{(i)} + \gamma_2^{(i)} - \gamma_1^{(i)}\gamma_2^{(i)}\} = \{0.9678, 0.7950, 0.4930\}$;
(2) $h_1 \dot{\otimes} h_2 = \bigcup_{i=1}^{3} \{\gamma_1^{(i)}\gamma_2^{(i)}\} = \{0.6622, 0.2950, 0.0770\}$;
(3) $2h_1 = \bigcup_{i=1}^{3} \{1 - (1 - \gamma^{(i)})^2\} = \{0.9471, 0.7500, 0.5775\}$;
(4) $h^2 = \bigcup_{i=1}^{3} \{(\gamma^{(i)})^2\} = \{0.5929, 0.2500, 0.1225\}$.

Based on the adjusted operational principle for HFEs, we investigate the improved HFPA operators as follows: