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New results on anti-synchronization of switched neural networks with time-varying delays and lag signals

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ABSTRACT

This paper investigates the problem of global exponential anti-synchronization of a class of switched neural networks with time-varying delays and lag signals. Considering the packed circuits, the controller is dependent on the output of the system as the inner states are very hard to measure. Therefore, it is necessary to investigate the controller based on the output of the neuron cell. Through theoretical analysis, it is obvious that the obtained ones improve and generalize the results derived in the previous literature. To illustrate the effectiveness, a simulation example with applications in image encryptions is also presented in the paper.

1. Introduction

With the rapid development of intelligent control, hybrid systems have been investigated extensively (Li & Cao, 2010; Li & Shen, 2010; Li & Song, 2013; Wen, Huang, Yu, Chen, & Zeng, 2016). As a special case of hybrid systems, switched systems consist of a family subsystems, which are controlled by a switching rule. In practical applications, many systems can be modeled as switched systems, switched circuits, switched networks, and so on. Considerable attention has been drawn to the theoretical analysis of switched systems (Huang, Qu, & Li, 2005).

Meanwhile, synchronization and anti-synchronization of neural networks have attracted great attention due to its great applications in many fields such as biological systems, secure communications, information science, image encryption, pseudo random number generator, and adaptive dynamic programmer (Chen & Dong, 1998; Chen, Su, & Chen, 2015; Chen, Zhang, Su, & Li, 2015; Chua & Roska, 2002; Hu, Yu, Jiang, & Teng, 2010; Huang, Li, Huang, & He, 2014; Luo, Sun, Zhang, & Cui, 2015; Wen, Zeng, Huang, & Meng, 2015; Wen, Zeng, Huang, & Zhang, 2014; Wu, Feng, & Lam, 2013; Wu, Shi, Su, & Chu, 2013; Zhang, Cui, & Luo, 2013; Zhang, Cui, Zhang, & Luo, 2010; Zhang, Liu, Luo, & Wang, 2013; Zhang, Ma, Huang, & Wang, 2010). Synchronization phenomena including complete synchronization (Kan, Wang, & Shu, 2013; Liang, Wang, Liu, & Liu, 2014; Shen, Wang, & Liu, 2010; Song, 2009; Tang, Wang, Gao, Qiao, & Kurths, 2014; Wen, Zeng, Huang, Yu, & Xiao, 2015; Xia, Yang, & Han, 2009; Zhu & Cao, 2011), generalized synchronization (Sun, Zeng, & Tian, 2010), phase synchronization (Breve, Zhao, Quiles, & Macau, 2009), lag synchronization (Yu & Cao, 2006) have been investigated. Meanwhile, anti-synchronization is a phenomenon that the state vectors of synchronized systems have the same absolute values but opposite signs. It is worth mentioning that in connected electronic networks, the occurrence of time delay is unavoidable because of finite signal transmission times, switching speeds, and some other reasons. Thus, the complete anti-synchronization of neural networks is hard to implement effectively and it is more reasonable to consider the lag synchronization problem.

Several control methods have been proposed for the lag synchronization of delayed neural networks, such as periodically intermittent control in Dan, Yang, and Feng (2013), Fang, Sun, Li, and Han (2013) and Huang, Li, Huang, Li, and Peng (2013). Exponential stability criteria are derived for the synchronization error systems with constant time delays in Dan et al. (2013) and Huang et al. (2013), however, these criteria are not applicable for systems with time-varying delays. Meanwhile, only asymptotical stability criteria are derived for synchronization error systems in Fang et al. (2013). Li and Bohnerb investigated the exponential

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synchronization of chaotic neural networks via LMI techniques in Li and Bohnerb (2010). However, there are few results on global exponential anti-synchronization of switched neural networks with time-varying delays and lag signals.

Motivated by the above discussion, this paper studies the problem of globally exponential anti-synchronization for switched neural networks with time-varying delays and lag signals. It is worth pointing out that, the proposed problem is non-trivial because of the difficulties such as that the controller is designed via the neuron activation function and lag signals.

The rest of the paper is organized as follows. In Section 2, a new model of lag anti-synchronization error system is formulated within a unified framework. In Section 3, lag anti-synchronization of switched neural networks is discussed by the controller based on the neuron activation function. Several sufficient conditions are derived to ensure the anti-synchronization of switched neural networks. Analysis has been made on results in this paper with respect to the previous ones. In Section 4, an illustrative example is discussed to demonstrate the effectiveness of the theoretical analysis. Finally, conclusions are drawn in Section 5.

2. Preliminaries

Denote $u = (u_1, \ldots, u_n)^T$, $|u|$ as the absolute-value vector; i.e., $|u| = (|u_1|, |u_2|, \ldots, |u_n|)^T$, $\|x\|_p$ as the p-norm of vector x, $1 \leq p < \infty$. $\|x\|_\infty = \max_{i\in\{1,2,\ldots,n\}} |x_i|$ is the infinity norm of vector x. Denote $\|D\|_p$ as the p-norm of the matrix D. Denote $\mathcal{C}$ as the set of continuous functions.

A set of neural networks is considered as the individual subsystems of the switched neural network. The driving switched neural network is described as follows:

$$\dot{x}(t) = -D_l x(t) + A_l g(x(t)) + B_l g(x(t-\tau(t))) + I, \tag{1}$$

where l is a switching signal taking its value in the finite set $\mathcal{I} = \{1, \ldots, N\}$, which means that the matrices (D_l, A_l, B_l) are allowed to take values in the finite set $\{(D_1, A_1, B_1), \ldots, (D_N, A_N, B_N)\}$. The parameters of system (1) are utilized to reflect the switched property of the electronic elements in neural networks, such as switched resistors and so on.

Throughout this paper, we assume that the switching rule l is known priori to the receiver and its instantaneous value is available in real time. The initial condition of system (1) is in the form of $x(t) = \phi(t) \in \mathcal{C}([-\chi, 0], \mathbb{R}^n)$, $\chi = \max\{\bar{\tau}\}$, where $\bar{\tau} = \max_{1\leq l\leq N} \bar{\tau}_l, 0 \leq \tau_l(t) \leq \bar{\tau}_l$.

Consider the following response system:

$$\dot{x}(t) = -D_l x(t) + A_l g(x(t)) + B_l g(x(t-\tau(t))) + I. \tag{2}$$

Define an indicator function $\Theta(t) = (\Theta_1(t), \ldots, \Theta_N(t))^T$, where

$$\Theta_l(t) = \begin{cases} 1, & \text{when the switched system is described by the } l\text{th mode } (D_l, A_l, B_l), \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

with $l = 1, \ldots, N$. Then the driving switched neural network (1) can be represented by

$$\dot{x}(t) = \sum_{l=1}^{N} \Theta_l(t)\Big(-D_l x(t) + A_l g(x(t)) + B_l g(x(t-\tau(t))) + I\Big). \tag{4}$$

It follows that $\sum_{l=1}^{N} \Theta_l(t) = 1$ under any switching rules. Assume that the response system has the same switching law as the driving system:

$$\dot{y}(t) = \sum_{l=1}^{N} \Theta_l(t)\Big(-D_l y(t) + A_l g(y(t)) + B_l g(y(t-\tau(t))) + I + u_l(t)\Big), \tag{5}$$

where $u_l(t)(l = 1, \ldots, N)$ are the controllers. The initial condition of system (5) is in the form of $y(t) = \varphi(t) \in \mathcal{C}([-\bar{\tau}, 0], \mathbb{R}^n)$.

In this paper, we assume the following:

A1. For $i \in \{1, 2, \ldots, n\}$, the activation function g_i is Lipschitz continuous; and $\forall r_1, r_2 \in \mathbb{R}$, there exists real number ι_i such that

$$0 \leq \frac{g_i(r_1) - g_i(r_2)}{r_1 - r_2} \leq \iota_i. \tag{6}$$

A2. For $i \in \{1, 2, \ldots, n\}$, $\tau_i(t)$ satisfies

$$0 \leq \tau_i(t) \leq \bar{\tau}_i, \qquad \dot{\tau}_i(t) \leq \mu_i < 1. \tag{7}$$

In order to derive sufficient conditions for the global exponential lag anti-synchronization of system (4) with system (5), we will need the following lemmas.

Lemma 1 (Zhao & Tan, 2007). *Given any real matrices X, Y, P of appropriate dimensions and a scalar $\epsilon_0 > 0$, where $P > 0$, the following inequality holds:*

$$X^TY + Y^TX \leq \epsilon_0 X^TPX + \epsilon_0^{-1} Y^TP^{-1}Y.$$

In particular, if X and Y are vectors, $X^TY \leq \frac{1}{2}(X^TX + Y^TY)$.

3. A new model for the anti-synchronization error system

In practice, it is hard to estimate real-time inner states of the integrated and packed circuit, the output of this circuit can be utilized to measure such packed circuit. Therefore, this paper aims to design a controller based on the output function, which is also called activation function in neuromorphic circuits to reach lag anti-synchronization ($y(t) \to -x(t-\xi)$ for some constant lag time $\xi > 0$). The error system can be obtained as

$$\dot{e}(t) = \sum_{l=1}^{N} \Theta_l(t)\Big(-D_l e(t) + A_l\Psi(e(t)) + B_l\Psi(e(t-\tau(t))) + u_l(t)\Big), \tag{8}$$

where $e(t) = (e_1(t), e_2(t), \ldots, e_n(t))^T$, is the lag anti-synchronization error, and

$$e_i(t) = y_i(t) + x_i(t-\xi_i). \tag{9}$$

The neural activation functions with/without delays are

$$\begin{aligned} \Psi(e(t)) &= (\Psi_1(e_1(t)), \ldots, \Psi_n(e_n(t)))^{\mathrm{T}} \\ &= g(e(t) + x(t-\xi)) + g(x(t-\xi)), \\ \Psi(e(t-\tau(t))) &= (\Psi_1(e_1(t-\tau_1(t))), \ldots, \Psi_n(e_n(t-\tau_n(t))))^{\mathrm{T}} \\ &= g(e(t-\tau(t)) + x(t-\tau(t)-\xi)) \\ &\quad + g(x(t-\tau(t)-\xi)). \end{aligned}$$

In the case of packed circuits, the controller is dependent on the output of the systems as follows

$$u_l(t) = K_l\Psi(e(t)), \tag{10}$$

where $K_l = (k_{lij})_{n\times n}$ is a constant gain matrix to be determined in order to achieve synchronization of the drive and response systems, $\Psi(e(t))$ is the output function without delays.

Based on controller (10), the error system (8) is transformed into

$$\dot{e}(t) = \sum_{l=1}^{N} \Theta_l(t)\Big(-D_l e(t) + \hat{A}_l\Psi(e(t)) + B_l\Psi(e(t-\tau(t)))\Big), \tag{11}$$

where $\hat{A}_L = (\hat{a}_{lij})_{n\times n} = (a_{lij} + k_{lij})_{n\times n}$.