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Generalized hesitant fuzzy sets and their application in decision support system

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ABSTRACT

Hesitant fuzzy sets are very useful to deal with group decision making problems when experts have a hesitation among several possible memberships for an element to a set. During the evaluating process in practice, however, these possible memberships may be not only crisp values in [0, 1], but also interval values. In this study, we extend hesitant fuzzy sets by intuitionistic fuzzy sets and refer to them as generalized hesitant fuzzy sets. Zadeh's fuzzy sets, intuitionistic fuzzy sets and hesitant fuzzy sets are special cases of the new fuzzy sets. We redefine some basic operations of generalized hesitant fuzzy sets, which are consistent with those of hesitant fuzzy sets. Some arithmetic operations and relationships among them are discussed as well. We further introduce the comparison law to distinguish two generalized hesitant fuzzy sets according to score function and consistency function. Besides, the proposed extension principle enables decision makers to employ aggregation operators of intuitionistic fuzzy sets to aggregate a set of generalized hesitant fuzzy sets for decision making. The rationality of applying the proposed techniques is clarified by a practical example. At last, the proposed techniques are devoted to a decision support system.

1. Introduction

Decision making problems referring to evaluating, prioritizing or selecting over some available alternatives are very common in practice [1]. Multiple criteria which may be conflicted with each other, or are conflicted between different levels of decision makers (DMs) (as can be seen in [2]), should be considered through the decision making process. There are generally two challenges in these problems. For example, a company wants to select a third party logistics supplier as a strategic partner. Several suppliers are taken into account for the final decision. The first challenge is caused by the complexity of the problem. One expert may be good at evaluating logistics capabilities of suppliers but weak in evaluating fixed assets. Thus the decision needs to be made by a group, or even multiple groups (as can be seen in [3]), of experts or DMs rather than individual DM. The other challenge is how to express preferences of DMs "accurately". Subjective and objective assessments provided by experts usually result in uncertain, imprecise, indefinite or subjective data [4]. One expert may express evaluations by linguistic terms, another may have hesitancy. In order to handle it, theories of probability and fuzzy mathematics have been expanded. Evidential reasoning theory [5] which acts as an extension of probability is a famous tool for decision making under uncertainty. But uncertainty is not probabilistic but rather imprecise or vague in many situations. Thus fuzzy logic and fuzzy set are popular when handling imperfect, vague or imprecise information.

Since it was introduced by Zadeh [6], theories of fuzzy sets serve as an excellent resolution of decision making under uncertainties. But the modeling tools of Zadeh's fuzzy sets (Z-FSs) are limited whereby two or more sources of vagueness appear simultaneously. Thus several generalizations and extensions of Z-FSs are developed, such as type-2 fuzzy sets [7,8], type-n fuzzy sets [8], intuitionistic fuzzy sets (IFSs) [9], fuzzy multisets [10] and hesitant fuzzy sets [11]. A type-2 fuzzy set enables us to define the membership of a given element in terms of a fuzzy set. As a generalization of type-2 fuzzy sets, type-n fuzzy sets, homoplastically, incorporate uncertainties in their memberships. IFSs or interval-valued fuzzy sets extend fuzzy sets by a hesitancy function, thus the membership takes the form of an interval. Fuzzy multisets allow elements repeated more than once in the set, thus can be seen as an extension of both Z-FSs and multisets. Torra recently defined hesitant fuzzy sets (abbreviated as T-HFSs) in which the membership is the union of several memberships of Z-FSs. T-HFSs are quite suit for the situation where we have a set of possible values, rather than a margin of error (as in IFSs) or some possibility distribution on the possible values (as in type-2 fuzzy sets). Torra [11] pointed out that it is useful to deal with all the possible values instead of considering just an aggregation operator.

There are some developments on T-HFSs. Torra and Narukawa [12] introduced the extension principle to apply it in decision

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making. Xia and Xu [13] developed a series of aggregation operators for hesitant fuzzy information and applied to multi-criteria decision making. Later, some induced aggregation operators in hesitant fuzzy setting are introduced by Xia et al. [14], the order inducing variables in which are defined by confident level of DMs. Based on Quasi arithmetic means, Xia et al. [15] discussed some ordered aggregation operators and induced ordered aggregation operators, as well as their application in group decision making. Some similarity measure and correlation measures are detailed studied in Xu and Xia [16,17], respectively.

In practice, we may have several possible memberships take the forms of both crisp values and interval values in [0,1] when discussing the membership of an element to a set. Suppose a decision organization with three groups of experts is authorized to assess the satisfactory degree of an alternative with respect to a criterion. Priorities of experts in each group are indifferent. In Group 1, some experts provide 0.5 surely, others provide 0.6 without hesitancy, and thus the assessment can be represented by a T-HFS {0.5, 0.6}. While in Group 2, some experts provide 0.4 doubtless, some argue between 0.45 and 0.55, and others insist on at least 0.6, then these three possible memberships can be represent by three IFSs (0.4, 1–0.4), (0.45, 1–0.55) and (0.6, 0), respectively. Group 3 provides between 0.5 and 0.7 consistently. An alternative resolution of this problem is that aggregating the information within each group at first and then aggregation the resultant information among groups. And the choices of aggregation operators usually depend on subjectivity of DMs. As can be seen in literatures [18–20], different operators may lead to different final decision. Therefore using aggregation operators twice (or even three times if multi criteria are considered in the example), which is common in group decision making, may lead to less robust decision. Table 1 shows two classes of rankings obtained by different times of aggregations. The problem and corresponding evaluation data can be found in [14]. It is clear that results of Class 2 are more confused and inconsistent with each other than that of Class 1. Additional, the process of aggregation is the average of original information by some means. For example, using the arithmetic average operator introduced in Xu [21], assessment of Group 2 can be resulted as (0.4905, 0). The employment of this mean to the second step of aggregation may lead to loss of information. Therefore we generalize the T-HFS to be fit for more general case. We allow each possible membership in the generalized hesitant fuzzy set includes hesitancy, in other words, the membership is the union of some IFSs or interval-valued fuzzy sets. There are mainly three advantages of the extension. First, as the case in T-HFSs, it is very useful to consider all possible memberships with hesitancy rather than considering just an aggregation operator. Second, it can eliminate times of using aggregation operators during the group decision making process, which can alleviate suffering from less robust decision led by times of aggregations. At last, individual expert can express his/her evaluations by either Z-FSs, IFSs, T-HFSs or the proposed fuzzy sets.

Table 1
Rankings obtained by different times of aggregations.

	Operator(s)	Ranking
Class 1	CIHFWA	$Y_1 \succ Y_4 \succ Y_5 \succ Y_2 \succ Y_3$
	$GCIHFWA_2$	$Y_1 \succ Y_5 \succ Y_4 \succ Y_2 \succ Y_3$
	$GCIHFWA_5$	$Y_1 \succ Y_5 \succ Y_4 \succ Y_2 \succ Y_3$
	$GCIHFWA_{-1}$	$Y_4 \succ Y_5 \succ Y_1 \succ Y_2 \succ Y_3$
	$GCIHFWA_{-2}$	$Y_4 \succ Y_1 \succ Y_5 \succ Y_2 \succ Y_3$
	$GCIHFWA_{-5}$	$Y_1 \succ Y_2 \succ Y_4 \succ Y_5 \succ Y_3$
Class 2	CIOWA + WA	$Y_2 \succ Y_1 \succ Y_4 \succ Y_5 \succ Y_3$
	CIOWA + GWA_{-5}	$Y_4 \succ Y_2 \succ Y_1 \succ Y_3 \succ Y_5$
	$GCIOWA_2$+WA	$Y_2 \succ Y_4 \succ Y_1 \succ Y_5 \succ Y_3$
	$GCIOWA_5$+WA	$Y_2 \succ Y_4 \succ Y_5 \succ Y_1 \succ Y_3$
	$GCIOWA_{-1}$+WA	$Y_1 \succ Y_2 \succ Y_4 \succ Y_5 \succ Y_3$
	$GCIOWA_{-2}$+WA	$Y_1 \succ Y_2 \succ Y_3 \succ Y_4 \succ Y_5$

CIHFWA: the confidence induced hesitant fuzzy weighted averaging operator in [13]; $GCIHFWA_\lambda$: the generalized confidence induced hesitant fuzzy weighted averaging operator in [13]; CIOWA: the confidence induced ordered weighted averaging operator in [13]; $GCIOWA_\lambda$: the generalized confidence induced ordered weighted averaging operator in [13]; WA: the weighted averaging operator; GWA_λ: the generalized weighted averaging operator. "A + B" means A is used in the first aggregation and B is used in the second aggregation.

In this study, therefore, we extend T-HFSs to generalized hesitant fuzzy sets (G-HFSs). Some basic operations on them are defined, such as union, intersection and some arithmetic operation on their elements. And their properties and relationships with T-HFSs are discussed as well. Then we develop a comparison law to distinguish information of two G-HFSs. A corresponding extension principle is introduced for further application to group decision making. To achieve it, the structure of the paper is as follows. Section 2 reviews some related preliminaries, such as T-HFSs and IFSs. In Section 3, G-HFSs are defined, some basic operations associated with their relationships are discussed, and comparison laws are developed as well. Section 4 presents the extension principle and Section 5 proposes a framework of G-HFSs based decision support system (DSS) and compares it with some other techniques. Then Section 6 concludes the paper.

2. Preliminaries

Due to the proposal of utilizing IFSs to generalize hesitant fuzzy sets, this section is devoted to recall some preliminaries involved in these two kinds of fuzzy sets.

2.1. Hesitant fuzzy sets

Sometimes, it is difficult to determine the membership of an element into a fixed set and which may be caused by a doubt among a set of different values. For the sake of a better description of this situation, Torra introduced the concept of T-HFSs as a generalization of fuzzy sets. The membership degree of a T-HFS is presented by several possible values in [0,1]. The definition is cited as follow.

Definition 1 [11]. Let X be a fixed set, then a hesitant fuzzy set (T-HFS) on X in terms of a function h is that when applied to X returns a subset of [0,1].

Furthermore, given a set of Z-FSs, a T-HFS could be defined in accordance with the union of their memberships as follow.

Definition 2 [11]. Given a set of N membership functions: $M=\{\gamma_1,\ldots,\gamma_N\}$, the T-HFS associated with M, that is h_M, is defined as follow:

$$h_M(x) = \cup_{\gamma\in M}\{\gamma(x)\}. \tag{1}$$

Xia and Xu [13] call $h_M(x)$ a hesitant fuzzy element (abbreviated as T-HFE) and H the set of all T-HFEs.

When decision information is represented by a collection of T-HFSs, it is necessary to introduce a function or mechanism to aggregate them for final decision making. Torra and Narukawa [12] proposed an extension principle which permits us to export operations on Z-FSs to T-HFSs as follow.

Definition 3 [12]. Let Θ be a function $\Theta{:}[0,1]^N \to [0,1]$, $H = \{h_1,h_2,\ldots,h_N\}$ be a set of T-HFSs on the reference set X. Then the extension of Θ on H is defined for each x in X by:

$$\Theta_H(x) = \cup_{\gamma\in\{h_1(x)\times h_2(x)\times\cdots\times h_N(x)\}}\{\Theta(\gamma)\}. \tag{2}$$

2.2. Intuitionistic fuzzy sets

IFSs introduced by Atanassov [9] have been proven to be highly useful to deal with uncertainty and vagueness. Hesitation of which