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Admissibility analysis for Takagi–Sugeno fuzzy singular systems with time delay ☆

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ABSTRACT

The issue of admissibility analysis for Takagi–Sugeno (T–S) fuzzy singular system with time delay is investigated in this paper. By adopting a novel tighter integral inequality, a sufficient delay-dependent criterion is built on the basis of strict linear matrix equalities (LMIs), which guarantees the admissibility of the considered system. A numerical example shows that the proposed method is efficient and less conservative.

1. Introduction

Singular systems, sometimes named as semi-state systems, generalized state-space systems, implicit systems, descriptor systems etc, have been studied broadly in the last years on account of their widespread applications in many fields, such as economic systems, power systems, large-scale systems, etc [1–3]. Compared with studies of standard state-space systems, the singular systems are much more complex to study, because apart from the stability, both the regularity and causality (for discrete singular systems) or absence of impulses (for continuous singular systems) should also be taken into consideration [4–6]. Moreover, as we all know, time delay frequently exists in many practical systems, for example, rolling mill systems, chemical engineering systems, etc [7–9]. In addition, time delay is an important source that often causes dynamic systems to be instable and poorly performed [10,11]. Hence, much attention has been paid to investigate the effect of time delay and how to cope with it, such as [12–14] etc. The issues of time-varying delay and mixed time delays of stochastic Markovian jump neural networks are addressed in [15] and [16], respectively.

Additionally, fuzzy logic plays a significant role in system modelling and data mining with characterising uncertainty [17–19]. The Takagi–Sugeno (T–S) fuzzy rule model is quite an efficient method to approximate the complicated nonlinear systems, which combines the mathematical theory and the fuzzy logic theory [20–22]. Based on "IF-THEN" rules, by "blending" every local linear system, the T–S fuzzy-model-based approach is able to handle nonlinearities existing in the application systems [23,24]. A lot of studies have been done on the research of T–S fuzzy systems [25–27]. On account of the interval T–S fuzzy approach, the issue of fuzzy control for nonlinear networked control systems with parameter uncertainties and packet dropouts is investigated in [28]. By utilizing delay partitioning approach, the dissipativity analysis is addressed for T–S fuzzy singular systems with constant time delay in [29]. For the interval time-varying delay, a variable delay decomposition approach is used to solve the issue of the stability of Takagi–Sugeno fuzzy systems in [30]. In [31], the issue of delay-dependent dissipative control of T–S fuzzy singular model with uncertainties and time delay is investigated. However, there exists ample room to improve the proposed results. A new integral inequality is devised to reduce the conservatism of linear systems with a discrete distributed delay in [32], which is much tighter than other existing integral inequalities. This greatly motivates us to extend this new integral inequality technique to singular systems and make the T–S fuzzy singular systems less conservative.

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This paper concerns the admissibility of T–S fuzzy singular systems with time delay by adopting a novel integral inequality, which uses a double integral of the system state and includes the Wirtinger-based inequality. Based on this inequality technique, a delay-dependent sufficient criterion is put forward to guarantee the admissibility of the considered system. The prime contribution of this paper is to decrease both the conservatism and computational burden compared with some existing results by using a novel integral inequality which is used for the first time to consider the admissibility of T–S fuzzy singular systems. Additionally, a good trade-off between these two performances is addressed. Finally, a numerical example demonstrates the advantages of this method to decrease the conservatism of the considered systems.

Notations: The notations are standard throughout this paper. $P>0$ shows that P is a symmetric and positive definite matrix. The superscript '-1' denotes the inverse of a matrix and 'T' denotes the transpose. $\mathbb{R}^n$ represents n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ means the set of all $n\times m$ real matrices. $\|\bullet\|$ denotes the Euclidean norm of a vector and sym $(A)=A+A^T$. '$*$' represents symmetric terms in a symmetric matrix. I stands for the identity matrix and 0 represents zero matrix with appropriate dimensions. Diag (X_1, X_2,…X_n) stands for a square matrix, in which X_i (a matrix or a number, $i=1,2,\ldots,n$) locates at the main diagonal in sequence and the other elements are all zero. Matrices are supposed to be with compatible dimensions if they are not mentioned explicitly.

2. Problem formulation

Consider a class of nonlinear singular system with time delay that is denoted by T–S fuzzy singular model as follows:

Plant Rule i: IF $\theta_1(t)$ is μ_{i1}; $\theta_2(t)$ is μ_{i2}; … and $\theta_p(t)$ is μ_{ip}, THEN

$$E\dot{x}(t)=A_ix(t)+A_{di}x(t-d),$$
$$x(t)=\phi(t), t\in[-d,0], i\in\mathbb{S}, \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector; $\theta(t)=[\theta_1(t),\ \theta_2(t),\ldots,\theta_p(t)]$ is the presumed variable; μ_{ij} is the fuzzy set; $\mathbb{S}=\{1,2,3,\ldots,r\}$ and r denotes the number of IF-THEN rules; E probably is singular and rank $(E)=g\le n; d$ is the known constant time delay satisfying $0<d_{\min}\le d\le d_{\max}$; $\phi(t)$ means a compatible vector-value initial function which satisfies certain strict consistent condition and guarantees a unique solution for any sufficiently differentiable input function [33]; A_i, A_{di} represent known real constant matrices.

Afterwards, the whole model of the above systems can be described by the following model:

$$E\dot{x}(t)=\sum\nolimits_{i=1}^{r}h_i(\theta(t))[A_ix(t)+A_{di}x(t-d)],$$
$$x(t)=\phi(t), t\in[-d,0], i\in\mathbb{S} \tag{2}$$

where

$$h_i(t)=\frac{\omega_i(\theta(t))}{\sum_{i=1}^{r}\omega_i(\theta(t))}, \omega_i(\theta(t))=\prod\nolimits_{j=1}^{p}\mu_{ij}(\theta(t)),$$

and $\mu_{ij}(\theta(t))$ denotes the grade of membership of $\theta_j(t)$ in μ_{ij}. Clearly, for all t, one can see

$$h_i(\theta(t))\ge 0, \sum\nolimits_{i=1}^{r}h_i(\theta(t))=1.$$

Definition 1 (*[34]*).

(i) If $\det(sE-\sum_{i=1}^{r}h_i(\theta(t))A_i)$ is not identically zero, the singular system (2) is called to be regular.
(ii) If $\deg(\det(sE-\sum_{i=1}^{r}h_i)\theta(t))A_i))=\text{rank}(E)$, the singular system (2) is called to be impulse-free.
(iii) If, for any $\varepsilon>0$, there exists a scalar $\delta(\varepsilon)>0$ such that, for any compatible initial conditions $\phi(t)$ satisfying $\sup_{-d_{max}\le t\le 0}\|\phi(t)\|<\delta(\varepsilon)$, the solution $x(t)$ of system (2) satisfies $\|x(t)\|<\varepsilon$ for $t\ge 0$, moreover, $\lim_{t\to+\infty}x(t)=0$, the singular system (2) is called to be asymptotically stable.
(iv) If the singular system (2) is regular, impulse-free and asymptotically stable, it is called to be admissible.

The following nomenclature is adopted to simplify vector and matrix symbolizations:

$$v_1(t)=\int_{t-d}^{t}x(s)ds, v_2(t)=\int_{t-d}^{t}\int_{t-d}^{s}x(u)duds,$$
$$\eta_1=\left[x^T(t)E^T\ v_1^T(t)E^T\ v_2^T(t)E^T\right]^T,$$
$$\xi(t)=\left[x^T(t)\ x^T(t-d)\ \frac{1}{d}v_1^T(t)E^T\ \frac{2}{d^2}v_2^T(t)E^T\right]^T,$$
$$e_i=\left[0_{n\times(i-1)n}\ I_n\ 0_{n\times(4-i)n}\right], i=1,2,3,4,$$
$$\Gamma_i=[A_i\ A_{di}\ 0_{n\times n}\ 0_{n\times n}], i\in\mathbb{S}.$$

Lemma 1. *Presume x as a differentiable function: $[\alpha,\beta]\longrightarrow\mathbb{R}^n$. For symmetric matrices $S\in\mathbb{R}^{n\times n}>0$, N_1, N_2, $N_3\in\mathbb{R}^{4n\times n}$ and $E\in\mathbb{R}^{n\times n}$ (rank $(E)=g\le n$), the following inequality holds:*

$$-\int_{\alpha}^{\beta}\dot{x}^T(s)E^TSE\dot{x}(s)ds\le\vartheta^T\Omega\vartheta, \tag{3}$$

where

$$\Omega=\tau\left(\tilde{E}^TN_1S^{-1}N_1^T\tilde{E}+\frac{1}{3}\tilde{E}^TN_2S^{-1}N_2^T\tilde{E}+\frac{1}{5}\tilde{E}^TN_3S^{-1}N_3^T\tilde{E}\right)$$
$$+\text{sym}\left(\tilde{E}^TN_1\Pi_1+\tilde{E}^TN_2\Pi_2+\tilde{E}^TN_3\Pi_3\right),$$
$$\Pi_1=E(e_1-e_2), \Pi_2=E(e_1+e_2)-2e_3, \Pi_3=E(e_1-e_2)-6e_3+6e_4,$$
$$\tilde{E}=\text{diag}(E,E,I,I), \hat{E}=\text{diag}(\tilde{E},\tilde{E},\tilde{E}),$$
$$\vartheta=\left[x^T(\beta)\ x^T(\alpha)\ \frac{1}{\tau}\int_{\alpha}^{\beta}(Ex(s))^Tds\ \frac{2}{\tau^2}\int_{\alpha}^{\beta}\int_{\alpha}^{s}(Ex(u))^Tduds\right]^T, \tau=\beta-\alpha.$$

Proof. Define

$$f_1(s)=\frac{2s-\beta-\alpha}{\beta-\alpha},$$
$$f_2(s)=\frac{6s^2-6(\beta+\alpha)s+\beta^2+4\beta\alpha+\alpha^2}{(\beta-\alpha)^2},$$
$$N=\left[N_1^T\ N_2^T\ N_3^T\right]^T,$$
$$\zeta(s)=\left[\vartheta^T\ f_1(s)\vartheta^T\ f_2(s)\vartheta^T\right]^T.$$

It is clear that

$$-2\zeta^T(s)\hat{E}^TNE\dot{x}(s)\le\zeta^T(s)\hat{E}^TNS^{-1}N\hat{E}\zeta(s)+\dot{x}^T(s)E^TSE\dot{x}(s). \tag{4}$$

Integrating (4) from α to β derives

$$-2\vartheta^T\tilde{E}^TN_1E(e_1-e_2)\vartheta-2\vartheta^T\tilde{E}^TN_2(Ee_1+Ee_2-2e_3)\vartheta$$
$$-2\vartheta^T\tilde{E}^TN_3(Ee_1-Ee_2-6e_3+6e_4)\vartheta$$
$$\le(\beta-\alpha)\vartheta^T\tilde{E}^TN_1S^{-1}N_1^T\tilde{E}\vartheta+\frac{(\beta-\alpha)}{3}\vartheta^T\tilde{E}^TN_2S^{-1}N_2^T\tilde{E}\vartheta$$
$$+\frac{(\beta-\alpha)}{5}\vartheta^T\tilde{E}^TN_3S^{-1}N_3^T\tilde{E}\vartheta+\int_{\alpha}^{\beta}\dot{x}^T(s)E^TSE\dot{x}(s)d(s). \tag{5}$$

Rearranging (5) derives (3), so the proof is completed. □

Remark 1. When $E=I$, Lemma 1 becomes into Lemma 1 in [32]. If rank $(E)=g\le n$, Lemma 1 can be used in singular systems, which shows that Lemma 1 has a wider application.

The prime object of this paper is to put forward a new admissibility criterion of the considered systems and reduce the conservatism of existing results by utilizing this new inequality technique in Lemma 1.