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Periodic solutions of stochastic coupled systems on networks with periodic coefficients

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ABSTRACT

This paper is concerned with periodic solutions of periodic stochastic coupled systems on networks. A systematic method of proving the existence of periodic solutions to the general stochastic coupled systems on networks is provided by using combined method of graph theory and Lyapunov method. Moreover, sufficient conditions for the existence of the periodic solutions to a type of stochastic coupled system on networks are established. In addition, based on Lyapunov method and graph theory, global asymptotic stability criterion for the periodic solution is also given. Finally, a numerical example is provided to illustrate the effectiveness of the results developed.

1. Introduction

In recent years, the theory of stochastic differential equations (SDEs) has attracted much attention since it is of practical importance, and has an important role in many ways such as in insurance, finance, population dynamics, control etc. [1]. There are many good books and papers which study SDEs and we here mention Arnold [2], Friedman [3], Has'minskii [4] and Mao [5,6] among others. Stability of SDEs has been well studied. For example, Mao [5] discussed several kinds of stochastic stabilities of SDEs and stochastic functional differential equations (SFDEs). Most of the existing papers on the stability of SDEs and SFDEs are mainly concerned with the stability of the equilibrium point. However, in the real world, most systems are in the periodic environment, and periodicity of dynamical systems is an interesting topic. But the existence and stability of periodic solution to the periodic stochastic differential equations are not well studied. Up to now, only a few papers and books about the periodic stochastic differential equations have been reported [4,7–10]. In [4], Hasminskii gave some basic results on the periodic solutions of SDEs. Using these results in [4], Zhang and Gopalsamy [7] studied two n-species stochastic population models with periodic coefficients. In papers [8,9], the authors studied the following periodic stochastic differential equation with finite delay or infinite delay

$$\mathrm{d}x(t)=f(t,x_t)\mathrm{d}t+g(t,x_t)\mathrm{d}B(t),$$

and gave some sufficient conditions for the existence of periodic solutions.

On the other hand, coupled systems on networks (CSNs) have been an active research area due to their applications in the fields such as artificial complex dynamical systems, neural networks, the spread of infectious diseases, biological fields and so on (see [11–17] and references therein). The dynamic properties, especially the stability, of CSNs have been widely studied in recent years, and many research papers have been reported (see Refs. [18–20, 23–28]). In paper [18], by using the results of graph theory, Li and Shuai successfully developed a systematic method to obtain the global asymptotical stability of CSNs. Paper [20] studied the global stability for stochastic coupled systems on networks (SCSNs). By using the method in [18], combining coincidence degree theory, Zhang and Li studied the existence and global exponential stability of the periodic solution to CSNs with time delay and neutral CSNs with delays [21,22]. The above papers [18,21,22] investigated the global stability of equilibria or the existence and global stability of the periodic solution to CSNs by using graph theory. As far as we know, the existence and global stability of the periodic solution to periodic stochastic coupled systems on networks have been rarely studied.

Motivated by above facts, in this paper, we mainly discuss the periodic solutions of SCSNs and establish a theoretical framework for the existence of periodic solutions to SCSNs. The major research methods are graph theory and Lyapunov method. As

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applications, we establish the sufficient principle for the existence and global asymptotic stability of periodic solution to a type of stochastic coupled system on networks. Finally, a numerical example is provided to illustrate the effectiveness of the theoretical results.

2. Main lemmas

For the sake of simplicity, we use the following notations in the sequel. Let $\mathbb{R}^+=[0,\infty)$ and $\mathbb{L}=\{1,2,\dots,l\}$. Denote by $|\cdot|$ the Euclidean norm for vectors. $C^{2,1}(\mathbb{R}^n\times\mathbb{R}^+;\mathbb{R}^+)$ denotes the space of all nonnegative functions which are twice continuously differentiable with respect to x and once continuously differentiable with respect to t. $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathbb{P})$ is a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. If $f(t)$ is a continuous bounded function on $[0,\infty)$, define

$$f^u=\sup_{t\in[0,\infty)} f(t),\quad f^l=\inf_{t\in[0,\infty)} f(t).$$

2.1. Graph theoretical results

In this paper, we will use the following basic lemma on graph theory. The concepts concerning graph theory are given in Appendix.

Lemma 1 (*Li and Shuai [18]*). *Assume $l\geq 2$. Let c_k denote the cofactor of the k-th diagonal element of Laplacian matrix of $(\mathcal{G},A)$. Then the following identity holds:*

$$\sum_{k,h=1}^{l} c_k a_{kh} F_{kh}(y_k,y_h)=\sum_{Q\in\mathbb{Q}} W(Q)\sum_{(s,r)\in E(C_Q)} F_{rs}(y_r,y_s).$$

Here $F_{rs}(y_r,y_s), 1\leq r,s\leq l$, are arbitrary functions, $\mathbb{Q}$ is the set of all spanning unicyclic graphs of $(\mathcal{G},A)$, $W(Q)$ is the weight of Q, and C_Q denotes the directed cycle of Q. In particular, if $(\mathcal{G},A)$ is strongly connected, then $c_k>0$ for $k=1,2,\dots,l$.

2.2. Periodic stochastic process

Lyapunov method plays an important role in the study of the existence of periodic solutions of SDEs. The following definitions and lemmas are taken from [4,9].

Definition 1. A stochastic process $x(t)$ is said to be periodic with period T if its finite dimensional distributions are periodic with period T, i.e., for any positive integer m and any moments of time $t_1,\dots,t_m$, the joint distributions of the random variables $x_{t_1+kT},\dots,x_{t_m+kT}$ are independent of k $(k=\pm 1,\pm 2,\cdots)$.

Lemma 2. *Consider the following periodic SDE*

$$dx(t)=b(x(t),t)dt+\sigma(x(t),t)dB(t), \tag{1}$$

where the coefficients $b:\mathbb{R}^n\times\mathbb{R}^+\to\mathbb{R}^n$, $\sigma:\mathbb{R}^n\times\mathbb{R}^+\to\mathbb{R}^{n\times m}$, $b(x,t+T)=b(x,t)$, $\sigma(x,t+T)=\sigma(x,t)$ for some $T>0$, and $B(t)$ is an m-dimensional Browian motion. Suppose that system (1) has the global solutions. Let $V(x,t)\in C^{2,1}(\mathbb{R}^n\times\mathbb{R}^+;\mathbb{R}^+)$ which is T-periodic in t, and satisfy the conditions

$$\inf_{\|x\|>l} V(x,t)\to\infty \text{ as } l\to\infty, \tag{2}$$

$$\inf_{\|x\|>l} LV(x,t)\to-\infty \text{ as } l\to\infty. \tag{3}$$

Then there exists a solution of (1) which is a T-periodic markov process, where

$$LV(x,t)=\left[\frac{\partial V}{\partial t}+\frac{\partial V}{\partial x}b+\frac{1}{2}\text{trace}(\sigma^T V_{xx}\sigma)\right](x,t),$$

where

$$\frac{\partial V}{\partial x}=\left(\frac{\partial V}{\partial x_1},\dots,\frac{\partial V}{\partial x_n}\right),\ V_{xx}=\left(\frac{\partial^2 V}{\partial x_i\partial x_j}\right)_{n\times n}.$$

For Eq. (1), we give several kinds of stochastic stabilities which can be found in [29].

Definition 2.

(i) The solution $x(t)=0$ is said to be stochastically stable if for every $\varepsilon>0$ and $s\geq t_0$

$$\lim_{x_0\to 0}\mathbb{P}\left(\sup_{[s,\infty)}\|x(t;s,x_0)\|\geq\varepsilon\right)=0.$$

(ii) $x(t)=0$ is stochastically asymptotically stable if, in addition,

$$\lim_{x_0\to 0}\mathbb{P}\left(\lim_{t\to\infty}\|x(t;s,x_0)\|=0\right)=1, s\geq t_0.$$

(iii) $x(t)=0$ is said to be globally asymptotically stable if, further,

$$\mathbb{P}\left(\lim_{t\to\infty}x(t;s,x_0)=0\right)=1,\ \forall x_0\in\mathbb{R}^n.$$

3. The existence of periodic solutions of stochastic coupled systems on networks

Based on graph theory, coupled systems can be described in a digraph. Now we demonstrate the construction of a stochastic coupled system on networks on a digraph $\mathcal{G}$ with l $(l\geq 2)$ vertices. Assume that each subsystem dynamics is described by

$$dx_k(t)=f_k(t,x_k(t))dt+g_k(t,x_k(t))dB_k(t),$$

where $x_k(t)=(x_{k1}(t),x_{k2}(t),\dots,x_{km}(t))^T$ represents the state of the k-th subsystem, the T-periodic functions f_k, g_k: $\mathbb{R}^+\times\mathbb{R}^m\to\mathbb{R}^m$ are the drift coefficient and the diffusion coefficient, respectively, $B_k(t)$ is one-dimensional Browian motion defined on $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathbb{P})$. We assume that the influence of the h-th subsystem on the k-th subsystem for the drift coefficient and the diffusion coefficient are described by $M_{kh}(t,x_h(t))$ and $N_{kh}(t,x_h(t))$, respectively. Here $M_{kk}=N_{kk}=0$ and $M_{kh}=N_{kh}=0$ if and only if there exists no influence from h-th subsystem to k-th subsystem. Then we can obtain an m-dimensional stochastic coupled system as follows:

$$dx_k(t)=\left[f_k(t,x_k(t))+\sum_{h=1}^{l}M_{kh}(t,x_h(t))\right]dt+g_k(t,x_k(t))dB_k(t)$$
$$+\sum_{h=1}^{l}N_{kh}(t,x_h(t))dB_h(t), k\in\mathbb{L}. \tag{4}$$

We assume that the coefficients of (4) are T-periodic in t. Stochastic coupled system (4) is constructed based on a digraph $\mathcal{G}$, in which every node of $\mathcal{G}$ represents a subsystem and the weight of arc (h,k) represents the influence of the h-th subsystem on the k-th subsystem. For example, for a digraph $\mathcal{G}$ with 4 vertices, the network among compartments can be depicted in Fig. 1.

For the aim of this paper, we assume that the functions f_k, M_{kh}, g_k and N_{kh} are such that system (4) has a unique global solution. In the following theorem, we will develop a systematic approach to obtain the existence of periodic solutions of system (4) by using graph theory and Lyapunov method.