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Brief Papers

# New delay-decomposing approaches to stability criteria for delayed neural networks ☆

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ABSTRACT

The problem of stability analysis for neural networks (NNs) with interval time-varying delay is investigated. New delay-decomposing approaches which are dividing the variation interval of the delay into two unequal subintervals are proposed. Some new simple Lyapunov–Krasovskii functionals (LKFs) are defined on the obtained subintervals. The integral inequality method and the reciprocally convex technique are utilized to deal with the derivative of the LKFs. Several improved delay-dependent criteria are derived in terms of the linear matrix inequalities (LMIs). Compared with some previous criteria, the proposed ones give the results with less conservatism and lower numerical complexity. Two numerical examples are included to illustrate the effectiveness and the improvement of the proposed method.


## 1. Introduction

In the past decades, NNs have been successfully applied in many fields such as pattern recognition, associative memories, signal processing, fixed-point computations, and so on [1–4]. In these applications, the key feature of the designed neural network keeps to be convergent and stability. On the other hand, time delays often occur in various NNs due to the finite switching speed of amplifiers or finite speed of information processing, and may cause undesirable dynamic network behaviors such as oscillation and instability. It is therefore that the stability analysis of various of NNs with kinds of delays has attracted considerable attention in recent years, e.g. see [5–15], and references therein. The existing stability criteria for delayed NNs can be classified into two cases, that is, the delay-independent conditions and the delay-dependent ones. In general, the delay-dependent stability criteria are less conservative delay-independent ones when the time delay is small. Thus in the past decade, the delay-dependent stability analysis on the delayed NNs has become an important topic of primary significance [16–25].

In the above-mentioned references, the delay for NNs can be categorized as two classes according to the lower bound of time-varying delay being 0 or large than 0. The main purpose of the existing references is to derive the maximum allowable upper bound on time-delay such that the designed delayed NNs keep to be globally stable in different ways. Some delay-dependent stability conditions for delayed NNs have been presented by using the free weighting matrix method in [17]. The main idea of the free weighting matrix method is to deal with the neglected integral term in relative literature by introducing slack variables, which leads to increase the numerical complexity. In fact, many methods were proposed to handle the bound of the integral terms. One of the well-used methods is in general called reciprocally convex method proposed by [18], see e.g. [14,15,21,23,26], and references therein. Zhang et al. [19] proposed a weighting-delay-based method to study the stability problem of a class of NNs with time-varying delay. In recent year, many works [13,16,21,24] focused on the application of delay-partitioning approach to deal with the stability analysis for continuous-time NNs with delay. By utilizing different free-weighting matrices in two delay subintervals, a piecewise delay method has been proposed for the stability analysis of delayed NNs in [20], which is similar to two delay-partitioning approach. By constructing an augmented LKFs which include triple-integral terms and by proposing a new activation function condition, some less conservative stability criteria have been established without using delay-partitioning method in [21]. In fact, the LKFs including triple-integral terms were also used in [22,23]. However,

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the augmented LKFs unavoidably lead to increase the numerical complexity. Very recently, some less conservative stability criteria with lower computational burden than relative references have been proposed based on the delay-partitioning approach in [25]. The main idea of the delay-partitioning method is divide the variation interval of the time delay into some equidistant subintervals to obtain more information of the delay. However, when the delay-partitioning number increases, the derived conditions become more complicated and computational burden grows bigger. Moreover, the maximum value of upper bound of the delay cannot be always obtained when the length of the subintervals is equal.

Motivated by this mentioned above, the aim of this work is to revisit the stability analysis for NNs with interval time-varying delays. The contribution of this paper lies in the following:

(1) New delay-decomposing approaches are proposed. Unlike the delay-partitioning method used in [13,16,21,24,25], the delay interval $[\tau_m, \tau_M]$ is divided into two unequal subintervals $[\tau_m, \tau_\alpha]$ and $[\tau_\alpha, \tau_M]$ in this work, where $\tau_\alpha = \alpha\tau_m + (1-\alpha)\tau_M$, $0<\alpha<1$, $\tau_m$ and $\tau_M$ are the lower bound and upper bound of the time-varying delay, respectively. Note that the range of the subinterval $[\tau_m, \tau_\alpha]$ is $(1-\alpha)(\tau_M - \tau_m)$, which decreases with the increase of the value of the parameter $\alpha$, while the range of the subinterval $[\tau_\alpha, \tau_M]$ is $\alpha(\tau_M-\tau_m)$, which decreases with the decrease of the value of the parameter $\alpha$. One can clearly see that the smaller range of the delay interval can be obtained, the more information of the time-varying $\tau(t)$ can be known. So, the merit of the proposed approaches lies that more information of the time-varying can be obtained by setting the value of the parameter $\alpha$.

(2) The presented stability criteria give the results with less conservatism and lower numerical complexity. Some simple LKFs are constructed on the obtained two subintervals, which make use of the more information of the delay and are with less numbers of decision variables than the ones used in [19–25]. The tighter upper bounds of the derivative of LKFs can be gotten by setting the parameter $\alpha$, using integral inequality method and reciprocally convex technique. Finally, two well-known numerical examples are given to demonstrate the effectiveness and less conservatism over the existing results.

Notation: In this paper, $\mathbb{R}^n$ denotes $n$-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. For symmetric matrices $X$ and $Y$, the notation $X>Y$(respectively, $X\geq Y$) means that the matrix $X-Y$ is positive definite (respectively, nonnegative). diag $\{\cdots\}$ denotes the block diagonal matrix. The subscript '$T$' denotes the transpose of the matrix. $I_n$ denotes the identity matrix.

## 2. Problem statements and preliminaries

Consider the delayed NNs with interval time-varying delay as follows:

$$\dot{z}(t) = -Cz(t) + Ag(z(t)) + Bg(z(t-\tau(t))) + u \tag{1}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector with $n$ neurons in the NNs, $u = [u_1, u_2, \ldots, u_n]^T \in \mathbb{R}^n$ represents a constant input vector, $g(z(t)) = [g_1(z(t)), g_2(z(t)), \ldots, g_n(z(t))]^T \in \mathbb{R}^n$ means the neuron activation function, $C = \text{diag}\{c_1, c_2, \ldots, c_n\} > 0$, and $A, B \in \mathbb{R}^{n\times n}$ are the connection weight matrix and the delayed connection weight matrix of appropriate dimensions. The time delay $\tau(t)$ is a time-varying continuous function satisfying

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M, \quad \dot{\tau}(t) \leq \mu \tag{2}$$

where $\tau_m, \tau_M, \mu$ are known constant scalar values. The activation functions $g_i(z_i(t)), i=1,2,\ldots,n$ are bounded and satisfy the following condition:

$$\rho_i^- \leq \frac{g_i(u)-g_i(v)}{u-v} \leq \rho_i^+, \quad \forall u,v \in \mathbb{R},\ u\neq v,\ i=1,2,\ldots,n \tag{3}$$

where $\rho_i^-$ and $\rho_i^+$ are constant values. Assume that $z^* = [z_1^*, z_2^*, \ldots, z_n^*]$ is an equilibrium point of the NNs (1) whose uniqueness has been reported by [12], by using the transformation $x(\cdot) = z(\cdot) - z^*(\cdot)$, NNs (1) can be shifted as follows:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) \tag{4}$$

where $x(\cdot) = [x_1(\cdot), \ldots, x_n(\cdot)]^T$, $f(x(t)) = [f_1(x_1(t)), \ldots, f_n(x_n(t))]^T$, $f_i(x_i(t)) = g_i(x_i(t)+z_i^*) - g_i(z_i^*)$ and $f_i(0)=0$ $(i=1,2,\ldots,n)$. The activation functions $f_i(\cdot)$ $(i=1,2,\ldots,n)$ satisfy the following condition:

$$\rho_i^- \leq \frac{f_i(u)-f_i(v)}{u-v} \leq \rho_i^+, \quad \forall u,v \in R,\ u\neq v,\ i=1,2,\ldots,n \tag{5}$$

which is equivalent to

$$[f_i(u)-f_i(v)-\rho_i^-(u-v)][f_i(u)-f_i(v)-\rho_i^+(u-v)] \leq 0. \tag{6}$$

If $v=0$ in (5), we can obtain

$$\rho_i^- \leq \frac{f_i(u)}{u} \leq \rho_i^+, \quad \forall u\neq 0,\ i=1,2,\ldots,n \tag{7}$$

which is equivalent to

$$[f_i(u)-\rho_i^- u][f_i(u)-\rho_i^+ u] \leq 0, \quad i=1,2,\ldots,n \tag{8}$$

The objective of this paper is to formulate the delay dependent stability conditions of system (4). The following lemmas will play important roles in deriving the criteria.

**Lemma 1** (*Park et al. [18]*). *Let $f_1, f_2, \ldots, f_N : \mathbb{R}^m \mapsto \mathbb{R}$ have positive values in an open subset $D$ of $\mathbb{R}^m$. Then, the reciprocally convex combination of $f_i$ over $D$ satisfies*

$$\min_{\{\alpha_i | \alpha_i>0, \sum_{\backslash\backslash hskip\backslash -i} \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i\neq j} g_{i,j}(t)$$

*subject to*

$$\left\{ g_{i,j} : R^m \to R, g_{j,i}(t) = g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{j,i}(t) & f_j(t) \end{bmatrix} \geq 0 \right\}.$$

**Lemma 2** (*Guo et al. [27]*). *For any matrices $Q>0, M, N, X$ with compatible dimensions, any continuous time vector function $x(t), \eta(t)$ with compatible dimensions and any scalar $\tau_M$ satisfying $0\leq\tau(t)\leq\tau_M$, the following integral equality holds:*

$$-\int_{t-\tau_M}^{t} x^{\mathrm{T}}(s)Qx(s)\,ds = \tau_M \eta^{\mathrm{T}}(t)X\eta(t)$$

$$+2\eta^{\mathrm{T}}(t)\left\{M\int_{t-\tau_M}^{t-\tau(t)} x(s)\,ds + N\int_{t-\tau(t)}^{t} x(s)\,ds\right\}$$

$$-\int_{t-\tau_M}^{t-\tau(t)} \begin{bmatrix} x(s) \\ \eta(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q & M^{\mathrm{T}} \\ M & X \end{bmatrix} \begin{bmatrix} x(s) \\ \eta(t) \end{bmatrix} ds$$

$$-\int_{t-\tau(t)}^{t} \begin{bmatrix} x(s) \\ \eta(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q & N^{\mathrm{T}} \\ N & X \end{bmatrix} \begin{bmatrix} x(s) \\ \eta(t) \end{bmatrix} ds.$$

**Remark 1.** In fact, the last two integral terms of the above integral equality can be ignored and an integral inequality obtained if

$$\begin{bmatrix} Q & M^{\mathrm{T}} \\ M & X \end{bmatrix} \geq 0$$

and

$$\begin{bmatrix} Q & N^{\mathrm{T}} \\ N & X \end{bmatrix} \geq 0.$$