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Brief Papers

Global exponential convergence of non-autonomous cellular neural networks with multi-proportional delays ☆

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ABSTRACT

The paper is concerned with the exponential convergence for a class of non-autonomous cellular neural networks with multi-proportional delays. By employing the differential inequality techniques, we establish a novel result to ensure that all solutions of the addressed system converge exponentially to zero vector. Our results complement with some recent ones. Moreover, an illustrative example and its numerical simulation are given to demonstrate the effectiveness of the obtained results.

1. Introduction

In recent years, the dynamical behaviors of delayed cellular neural networks (DCNNs) have been receiving much attention due to their potential applications in associated memory, parallel computing, pattern recognition, signal processing and optimization problems (see [1–6]). In particular, to control the networks running time according to the network allowed delays, the proportional delay is one of the many objective-existent delay types such as the proportional delay usually is required in web quality of service routing decision [7]. Moreover, the systems with proportional delays have been used to model various problems in biology, economy, control, electrodynamics, and so on. To name a few, we refer the readers to [8–12] and the references cited therein.

As is well known, since the exponential convergent rate can be unveiled, there have been extensive results on the problem of the exponential convergence of delayed neural networks models in the literature. We refer the reader to [13–18] and the references cited therein. Furthermore, by means of the transformation $y(t)=x(e^t)$, the exponential stability of the cellular neural networks (CNNs) systems with the proportional delays have been extensively and intensively studied in [19–23]. Consequently, the authors in [19–23] obtain that exponential convergence of $y(t)$. Clearly, as pointed out in Remark 3 of [11], when $y(t)=e^{-t}$, then $x(t)=x(e^{\ln t})=y(\ln t)=\frac{1}{t}$. This implies that $x(t)=y(\ln t)=e^{-\ln t}$ is the different exponential convergence from the case that $x(t)=O(e^{-t})$.

On the other hand, during the past few years, by introducing a new dynamical system with unbounded time-varying delays, the authors in [24–26] obtained a lot of important and interesting results on stability of CNNs with constant leakage coefficients. Meanwhile, many scholars in [19–23] have paid much attention to the convergence on CNNs under the condition that the leakage term coefficient function is not oscillating. Most recently, as pointed out in [16–18], CNNs with oscillating leakage term coefficients has more realistic significance. It is worth mentioning that, computing the upper right derivative of the Lyapunov function is key proof method for convergence of CNNs in [19–23], which is invalid for CNNs with oscillating leakage term coefficients. This motivates us to further study the exponential convergence of CNNs with oscillating leakage term coefficients and involving the proportional delays.

In this paper, without assuming that the leakage term coefficient function is not oscillating, we consider the following class of non-autonomous CNNs with multi-proportional delays:

$$\begin{cases} \dot{x}_i(t)=-c_i(t)x_i(t)+\sum_{j=1}^{n}a_{ij}(t)f_j(x_j(t))+\sum_{j=1}^{n}b_{ij}(t)g_j(x_j(q_{ij}t))+I_i(t), & t\geq 1, \\ x_i(s)=\varphi_i(s), s\in[\rho_i,\ 1], \quad i=1,2,\dots,n \end{cases} \tag{1.1}$$

where n corresponds to the number of units in a neural network, $x_i(t)$ corresponds to the state vector of the ith unit at the time t,

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$c_i(t)$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at the time t; $a_{ij}(t)$ and $b_{ij}(t)$ denote the strengths of connectivity between the cells j and i at time t and $q_{ij}t$, respectively; $f_i(\cdot)$ and $g_i(\cdot)$ denote the nonlinear continuous activation functions; $I_i(t)$ denotes the external inputs at time t; $q_{ij}, i, j \in J = \{1, 2, \ldots, n\}$ are proportional delay factors and satisfy $0 < q_{ij} \le 1$, and $q_{ij}t = t-(1-q_{ij})t$, in which $\tau_{ij}(t) = (1-q_{ij})t$ is the transmission delay function, and $(1-q_{ij})t \to +\infty$ as $q_{ij} \ne 1$, $t \to +\infty$; $\varphi_i(s)$ denotes the initial value of $x_i(s)$ at $s \in [\rho_i, \ 1]$, $\rho_i = \min_{1 \le j \le n}\{q_{ij}\}$, and $\varphi_i \in C([\rho_i, \ 1], \mathbb{R})$.

For convenience, we denote by $\mathbb{R}^n(\mathbb{R} = \mathbb{R}^1)$ the set of all n-dimensional real vectors (real numbers). For any $x = (x_1, \ x_2, \cdots, x_n)^T \in \mathbb{R}^n$, we let $|x|$ denote the absolute-value vector given by $|x| = (|x_1|, |x_2, \ldots, |x_n|)^T$, and define $\|x\| = \max_{i \in J}|x_i|$. Throughout this paper, it will be assumed that $c_i, I_i, \ a_{ij}, \ b_{ij} : \mathbb{R} \to \mathbb{R}$ are bounded and continuous functions, where $i, \ j \in J$.

We also make the following assumptions which will be used later.

(H_0) for each $i \in J$, there exist positive constants c_i^* and K such that

$$e^{-\int_s^t c_i(u)du} \le K e^{-(t-s)c_i^*}, \quad \text{for all } t, s \in \mathbb{R} \text{ and } t-s \ge 0.$$

(H_1) there exist nonnegative constants L_j^f and L_j^g such that

$$|f_j(u)| \le L_j^f|u|, |g_j(u)| \le L_j^g|u|, \quad \text{for all } u \in \mathbb{R}, \ j \in J.$$

(H_2) for each $i \in J$, there exist positive constants $\xi_1, \xi_2, \ldots, \xi_n$ and λ_0 such that

$$\sup_{t \ge 1}\{-c_i^* + KF_i(t)\} < 0 \quad \text{and} \quad I_i(t) = O(e^{-\lambda_0 t}) \text{ as } t \to +\infty,$$

where

$$F_i(t) = \xi_i^{-1}\sum_{j=1}^n |a_{ij}(t)|L_j^f\xi_j + \xi_i^{-1}\sum_{j=1}^n |b_{ij}(t)|L_j^g\xi_j e^{\lambda_0(1-q_{ij})t}.$$

2. Global exponential convergence

Theorem 2.1. *Let (H_0), (H_1) and (H_2) hold. Then, for every solution $x(t)$ of system (1.1), there exists a positive constant λ such that*

$$x_i(t) = O(e^{-\lambda t}) \quad \text{as } t \to +\infty, \ i \in J.$$

Proof. Suppose that $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is an arbitrary solution of (1.1) associated with initial value $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)^T$ satisfying the second equation of (1.1).

Let

$$y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T = (\xi_1^{-1}x_1(t), \xi_2^{-1}x_2(t), \ldots, \xi_n^{-1}x_n(t))^T.$$

Then

$$y_i'(t) = -c_i(t)y_i(t) + \xi_i^{-1}\sum_{j=1}^n a_{ij}(t)f_j(x_j(t)) + \xi_i^{-1}\sum_{j=1}^n b_{ij}(t)g_j(x_j(q_{ij}t)) + \xi_i^{-1}I_i(t), \quad i \in J. \tag{2.1}$$

Define a continuous function $\Gamma_i(\omega)$ by setting

$$\Gamma_i(\omega) = \sup_{t \ge 1}\{\omega - c_i^* + K[F_i(t) + \omega]\}, \quad \text{where } \omega \in [0, \ \lambda_0], \ i \in J.$$

Then, from (H_2), we have

$$\Gamma_i(0) = \sup_{t \ge 1}\{-c_i^* + KF_i(t)\} < 0, \quad i \in J,$$

which, together with the continuity of $\Gamma_i(\omega)$, implies that we can choose a constant $\lambda \in (0, \min\{\lambda_0, \min_{i \in J} c_i^*\})$ such that

$$\Gamma_i(\lambda) = \sup_{t \ge 1}\{\lambda - c_i^* + K[F_i(t) + \lambda]\} < 0, \tag{2.2}$$

and

$$\begin{aligned}
&\sup_{t \ge 1}\left\{\lambda - c_i^* + K\left[\xi_i^{-1}\sum_{j=1}^n |a_{ij}(t)|L_j^f\xi_j + \xi_i^{-1}\sum_{j=1}^n |b_{ij}(t)|L_j^g\xi_j e^{\lambda(1-q_{ij})t} + \lambda\right]\right\} \\
&\le \sup_{t \ge 1}\left\{\lambda - c_i^* + K\left[\xi_i^{-1}\sum_{j=1}^n |a_{ij}(t)|L_j^f\xi_j + \xi_i^{-1}\sum_{j=1}^n |b_{ij}(t)|L_j^g\xi_j e^{\lambda_0(1-q_{ij})t} + \lambda\right]\right\} \\
&= \sup_{t \ge 1}\{\lambda - c_i^* + K[F_i(t) + \lambda]\} < 0, \ i \in J.
\end{aligned} \tag{2.3}$$

Let

$$\|\varphi\|_\xi = \max_{1 \le i \le n}\left\{\xi_i^{-1}\max_{t \in [\rho_i, \ 1]}|\varphi_i(t)|\right\}. \tag{2.4}$$

For any $\varepsilon > 0$, we obtain

$$|y_i(t)| < (\|\varphi\|_\xi + \varepsilon)e^{-\lambda(t-1)} < M(\|\varphi\|_\xi + \varepsilon)e^{-\lambda(t-1)} \text{ for all } t \in [\rho_i, \ 1],$$

and

$$\|y(t)\| < (\|\varphi\|_\xi + \varepsilon)e^{-\lambda(t-1)} < M(\|\varphi\|_\xi + \varepsilon)e^{-\lambda(t-1)} \quad \text{for all } t \in \left[\max_{1 \le i \le n}\rho_i, \ 1\right],$$

where $M > K+1$ is a sufficiently large constant such that

$$|\xi_i^{-1}I_i(t)| < \lambda M(\|\varphi\|_\xi + \varepsilon)e^{-\lambda(t-1)} \quad \text{for all } t \ge 1, i \in J. \tag{2.5}$$

In the following, we will show

$$\|y(t)\| < M(\|\varphi\|_\xi + \varepsilon)e^{-\lambda(t-1)} \quad \text{for all } t > 1. \tag{2.6}$$

Otherwise, there must exist $i \in \{1, 2, \ldots, n\}$ and $\theta > 1$ such that

$$\|y(\theta)\| = |y_i(\theta)| = M(\|\varphi\|_\xi + \varepsilon)e^{-\lambda(\theta-1)}, \tag{2.7}$$

and

$$|y_j(t)| < M(\|\varphi\|_\xi + \varepsilon)e^{-\lambda(t-1)} \quad \text{for all } t \in [\rho_j, \theta), \ j \in J. \tag{2.8}$$

Note that

$$y_i'(s) + c_i(s)y_i(s) = \xi_i^{-1}\sum_{j=1}^n a_{ij}(s)f_j(x_j(s)) + \xi_i^{-1}\sum_{j=1}^n b_{ij}(s)g_j(x_j(q_{ij}s)) + \xi_i^{-1}I_i(s), \ s \in [1, \ t], \quad t \in [1, \theta]. \tag{2.9}$$

Multiplying both sides of (2.9) by $e^{\int_1^s c_i(u)du}$, and integrating it on $[1, \ t]$, we get

$$y_i(t) = y_i(1)e^{-\int_1^t c_i(u)du} + \int_1^t e^{-\int_s^t c_i(u)du}\left[\xi_i^{-1}\sum_{j=1}^n a_{ij}(s)f_j(x_j(s)) + \xi_i^{-1}\sum_{j=1}^n b_{ij}(s)g_j(x_j(q_{ij}s)) + \xi_i^{-1}I_i(s)\right]ds, \quad t \in [1, \ \theta].$$

Thus, with the help of (2.3), (2.5) and (2.8), we have

$$\begin{aligned}
|y_i(\theta)| &= \left|y_i(1)e^{-\int_1^\theta c_i(u)du} + \int_1^\theta e^{-\int_s^\theta c_i(u)du}\left[\xi_i^{-1}\sum_{j=1}^n a_{ij}(s)f_j(x_j(s)) + \xi_i^{-1}\sum_{j=1}^n b_{ij}(s)g_j(x_j(q_{ij}s)) + \xi_i^{-1}I_i(s)\right]ds\right| \\
&\le (\|\varphi\|_\xi + \varepsilon)Ke^{-c_i^*(\theta-1)} + \int_1^\theta Ke^{-(\theta-s)c_i^*}\left|\xi_i^{-1}\sum_{j=1}^n a_{ij}(s)f_j(x_j(s))\right.
\end{aligned}$$