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The existence of periodic solutions for coupled Rayleigh system

Ying Guo [a,b], Shuang Liu [a], Xiaohua Ding [a,*]

[a] Department of Mathematics, Harbin Institute of Technology (Weihai), Weihai 264209, PR China
[b] Department of Mathematics, Heilongjiang Bayi Agricultural University, Daqing 163319, PR China

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ABSTRACT

This paper is concerned with the existence of periodic solutions for coupled Rayleigh system (CRS). A sufficient criterion for the existence of periodic solutions for CRS is provided via an innovative method of combining graph theory with coincidence degree theory as well as Lyapunov method. As a subsequent result, coupled Lord Rayleigh system is also discussed. Subsequently, a sufficient condition is given to determine the existence of its periodic solutions. Finally, a numerical example and its simulations are presented to illustrate the effectiveness and feasibility of the proposed criterion.

1. Introduction

An important class of Rayleigh system is described by the following form:

$$x''(t)+f(t,x'(t))+g(t,x(t))=e(t), \tag{1}$$

where $f,g:\mathbb{R}\times\mathbb{R}\rightarrow\mathbb{R}$ and $e:\mathbb{R}\rightarrow\mathbb{R}$ are continuous functions. The dynamic behaviors of system (1) have been an active research topic due to their extensive applications in physics, mechanics, engineering technique, and other areas (see [1–4] and the references therein). It has been shown that these successful applications are greatly dependent on the existence of periodic solutions for system (1). In [5], for example, the periodicity of system (1) plays an important role in modeling the oscillations of a clarinet reed. Moreover, the periodicity analysis of system (1) has been a subject of intense activities, and many results have been reported. See, to name a few, [4,6–8].

In fact, as a main property of dynamic systems, periodicity has received a range of interest among scholars from miscellaneous research fields, owing to its universal existence in biological systems, electronic systems and neural networks. In more detail, a number of useful sufficient criteria about the existence of periodic solutions for some biological systems have been established in [9–11] and some results about the periodicity of neural networks have also been reported in [12–14]. In the above cited works, many theorems and methods have been applied to research the existence of periodic solutions, such as various fixed point theorems, the continuation theorem of coincidence degree theory, Lyapunov method, and so on.

With the popularity of coupled systems, the dynamics of coupled systems on networks (CSN) have gained increasing recognition in understanding numerous natural systems in mechanical, electronic, and biological fields. Also, a significant number of publications concerned with the dynamics of CSN have been produced in [15–19]. It is worth pointing out that a graph-theoretic approach was applied to explore the dynamic properties of CSN by Li et al. [19,20]. From then on, a plenty of researchers devoted themselves to this method for CSN and many novel results were presented [21–29]. In general, since the dynamics of CSN depend not only on the individual vertex dynamics but also on the coupling topology, the research of CSN is very difficult. Therefore, so far the periodicity of CSN has rarely been studied, and we here mention only [30,31]. In addition, to the best of authors' knowledge, while coincidence degree theory provides a powerful tool to study the existence of periodic solutions, it is of great difficulty to research coupled systems using such a single method because of coupled systems' large dimensions.

* Corresponding author. Tel.: +86 0631 5672718.
E-mail addresses: mathdxh@126.com, mathdxh@hit.edu.cn (X. Ding).

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Considering the facts above, we study the existence of periodic solutions for coupled Rayleigh system (CRS) in this paper. The main purpose of this paper is to provide a sufficient condition to guarantee the existence of periodic solutions for CRS by a new method of combining graph theory with coincidence degree theory and Lyapunov method. It is worth pointing out that the method of incorporating coincidence degree theory and Lyapunov method to investigate the existence of periodic solutions for some systems has been used in [10,11,30,31]. However, we can find easily from these references that it is rather difficult to estimate the priori bounds of unknown solutions to operation equation $Lx=\lambda Nx$ for a specific system only by analysis techniques, let alone for complicated CSN caused by dimension. In this paper, CRS is described by a directed graph on the basis of graph theory, where system (1) is assigned at each vertex which is called vertex system, and the interconnections and interactions among vertex systems are characterized as directed arcs. We overcome the above problem by the means of graph theory and Lyapunov method. And based on coincidence degree theory, an easily verifiable criterion for the existence of periodic solutions for CRS is derived. Here our interest focuses on the relation between the existence of periodic solutions for CRS and the topology structure of the digraph. Compared with some recent results in [10,11,30,31], the chief contributions of our study are as follows:

1. An innovative method based on graph theory, coincidence degree theory, and Lyapunov method is exploited to discuss the existence of periodic solutions for CRS.
2. The result of this paper shows that the existence of periodic solutions for CRS relates closely to the topology structure of the digraph.

The remainder of this paper is organized as follows: in Section 2, preliminaries and problem statement are presented; in Section 3, based on graph theory, coincidence degree theory, and Lyapunov method, the sufficient criterion for the existence of periodic solutions for CRS is obtained. Ultimately, a numerical example and its simulations are presented in Section 4 to show the effectiveness of our theoretical result.

2. Modeling and preliminaries

In this section, some useful notations, lemmas, and preliminaries about graph theory and coincidence degree theory will be introduced for convenience.

2.1. Notations

Let $\mathbb{R}$ and $\mathbb{R}^n$ be the set of real numbers and n-dimensional Euclidean space, respectively. Let $\mathbb{R}^+=[0,+\infty)$, $\mathbb{L}=1,2,\ldots,l$, and $|\cdot|$ denote the Euclidean norm for vectors. The superscript symbol "T" represents the transpose of a vector. Write $C^1(\mathbb{R}^n;\mathbb{R}^+)$ for the family of all nonnegative functions $V(x)$ on $\mathbb{R}^n$ that are continuously once differentiable.

2.2. Graph theory

The following basic concepts and an important result on graph theory can be found in [35] and [19], respectively. A digraph $\mathcal{G}=(\mathbb{L},E)$ contains a set $\mathbb{L}$ of vertices and a set E of arcs (k,h) leading from initial vertex k to terminal vertex h. A subgraph $\mathcal{H}$ of $\mathcal{G}$ is said to be spanning if $\mathcal{H}$ and $\mathcal{G}$ have the same vertex set. A digraph $\mathcal{G}$ is weighted if each arc (h,k) is assigned a positive weight a_{kh}. Here $a_{kh}>0$ if and only if there exists an arc from vertex h to vertex k in $\mathcal{G}$, and we call $A=(a_{kh})_{l\times l}$ as the weight matrix. The weight $W(\mathcal{G})$ of $\mathcal{G}$ is the product of the weights on all its arcs. A directed path $\mathcal{P}$ in $\mathcal{G}$ is a subgraph with distinct vertices $\{i_1,i_2,\ldots,i_s\}$ such that its set of arcs is $\{(i_k,i_{k+1}):k=1,2,\ldots,s-1\}$. If $i_s=i_1$, we call $\mathcal{P}$ a directed cycle. A connected subgraph $\mathcal{T}$ is a tree if it contains no cycles. A tree $\mathcal{T}$ is rooted at vertex k, called the root, if k is not a terminal vertex of any arcs, and each of the remaining vertices is a terminal vertex of exactly one arc. A subgraph $\mathcal{Q}$ is unicyclic if it is a disjoint union of rooted trees whose roots form a directed cycle. A digraph $\mathcal{G}$ is strongly connected if, for any pair of distinct vertices, there exists a directed path from one to the other. Denote the digraph with weight matrix A as $(\mathcal{G},A)$. A weighted digraph $(\mathcal{G},A)$ is said to be balanced if $W(\mathcal{C})=W(-\mathcal{C})$ for all directed cycles $\mathcal{C}$. Here, $-\mathcal{C}$ denotes the reverse of $\mathcal{C}$ and is constructed by reversing the direction of all arcs in $\mathcal{C}$. For a unicyclic graph $\mathcal{Q}$ with cycle $\mathcal{C}_\mathcal{Q}$, let $\tilde{\mathcal{Q}}$ be the unicyclic graph obtained by replacing $\mathcal{C}_\mathcal{Q}$ with $-\mathcal{C}_\mathcal{Q}$. Suppose that $(\mathcal{G},A)$ is balanced, then $W(\mathcal{Q})=W(\tilde{\mathcal{Q}})$. The Laplacian matrix of $(\mathcal{G},A)$ is defined as

$$\begin{pmatrix} \sum_{k\neq 1}a_{1k} & -a_{12} & \cdots & -a_{1l} \\ -a_{21} & \sum_{k\neq 2}a_{2k} & \cdots & -a_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{l1} & -a_{l2} & \cdots & \sum_{k\neq l}a_{lk} \end{pmatrix}.$$

Lemma 1. *(Li et al. [19]) Assume $l\geq 2$. Let c_k denote the cofactor of the kth diagonal element of Laplacian matrix of $(\mathcal{G},A)$. Then the following identity holds:*

$$\sum_{k,h=1}^{l} c_k a_{kh} F_{kh}(y_k,y_h)=\sum_{Q\in\mathbb{Q}} W(Q)\sum_{(s,r)\in E(C_Q)} F_{rs}(y_r,y_s).$$

Here $F_{rs}(y_r,y_s)$, $1\leq r,s\leq l$, are arbitrary functions, $\mathbb{Q}$ is the set of all spanning unicyclic graphs of $(\mathcal{G},A)$, $W(Q)$ is the weight of Q and C_Q denotes the directed cycle of Q. In particular, if $(\mathcal{G},A)$ is strongly connected, then $c_k>0$ for $k\in\mathbb{L}$.

2.3. Coincidence degree theory

In this subsection, for the sake of the reader's convenience, we shall first summarize below some concepts and results from [36]. Let X and Z be normed vector spaces, $L:\mathrm{Dom}L\subset X\to Z$ be a linear mapping, $N:X\to Z$ be a continuous mapping. The mapping L is called a Fredholm mapping of index zero if $\dim \mathrm{Ker}L=\mathrm{Codim}\ \mathrm{Im}L<+\infty$ and $\mathrm{Im}L$ is closed in Z. If L is a Fredholm mapping of index zero, and there