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Leader-following consensus of data-sampled multi-agent systems with stochastic switching topologies

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ABSTRACT

This paper investigates the leader-following consensus problem for multi-agent systems with Markovian switching topologies in a sampled-data setting. We study two algorithms corresponding to the case where the leader's state is time varying or time invariant. With a time-varying leader's state, we present necessary and sufficient conditions for boundedness of the tracking error systems. With a time-invariant leader's state, we present necessary and sufficient conditions for mean-square stability of the tracking error systems. An optimization algorithm is given to derive the allowable control gains or the feasible sampling period. Simulation examples are presented to show the usefulness of the results.

1. Introduction

Leader-following consensus has received significant attention in the control field in recent years. This is mainly due to its wide applications in engineering, such as unmanned air vehicles, mobile robotic swarms, wireless sensor networks and cooperative surveillance [1–5]. The main idea of leader-following consensus is that the leader sends its state information to the followers directly or indirectly such that the tracking errors between leader and all followers are as small as possible. There are many publications on the topic of the leader-following consensus problem [6–8]. In [6], the multi-agent system considered was with measurement noises and directed interconnection topology. A sufficient condition for mean-square stability of the closed loop tracking control system was obtained by designing distributed estimators. In [7], both leaderless and leader-following consensus problems were studied. The stability or boundedness conditions were presented based on Lyapunov theorems and Nyquist stability criterion. By using the sampled-data control approach, the leader-following consensus for multi-agent systems was studied in [8]. The topology considered is deterministic.

Data-sampled approach is frequently used to discretize the continuous-time system in control community. In recent years this method is also used to study the multi-agent systems [8–12]. Two sampled-data coordination algorithms for double-integrator dynamics were studied in [10] where the interaction topology is fixed undirected/directed. In [11], the consensus problem of double-integrator multi-agent systems with both fixed and switching topologies was studied. The switching signal is arbitrary and only a sufficient condition is derived to solve a consensus problem in this case. In [12], the authors researched the stochastic bounded consensus tracking problems of multi-agent systems, where the sampling delay induced by the sampling process was considered.

The topologies in the above literature are all deterministic or switching in a deterministic framework. However, the system models are sometimes switching stochastically due to the internal or/and external disturbance. Similar to some other control systems, the Markovian switching model has been used to describe the interaction topology among the agents in very recent years [13,16]. In [13], the static stabilization problem of a decentralized discrete-time single-integrator network with Markovian switching topologies was studied. In [14], the authors considered the consensus for a network of single-integrator agents with Markovian switching topologies. In [15], the authors studied the mean-square consentability problem for a network of double-integrator agents with Markovian switching topologies. In [16], the authors studied the distributed discrete-time coordinated tracking problem for multi-agent systems with Markovian switching topologies in case of the transition probabilities are equal.

Motivated by the former considerations, we will extend the leader-following consensus problem in [8] to the case of Markovian switching topologies in this paper. In this case, the leader-following consensus problem will become more challenging. Both time-invariant and time-varying leader are considered. Based on algebra graph theory and Markovian jump system theory, we present the necessary and sufficient conditions for the convergence of the tracking error systems. An optimization algorithm

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will be given to derive the allowable control gains and sampling period.

Notation: Let $\mathbb{R}$ and $\mathbb{N}$ represent, respectively, the real number set and the non-negative integer set. Denote the spectral radius of the matrix M by $\rho(M)$. Suppose that $A, B \in \mathbb{R}^{p\times p}$. Let $A \succcurlyeq B$ (respectively, $A \succ B$) denote that $A-B$ is symmetric positive semi-definite (respectively, symmetric positive definite). Denote the determinant of the matrix A by $\|A\|$. Given $X(k)\in\mathbb{R}^p$, define $\|X(k)\|_E \triangleq \|E[X(k)X^T(k)]\|_2$, where $E[\cdot]$ is the mathematical expectation. "$\otimes$" represents the Kronecker product of matrices. I_n denotes the $n\times n$ identity matrix. Let $\mathbf{1}_n$ and $\mathbf{0}_{m\times n}$ denote, respectively, the $n\times 1$ column vector with all components equal to 1 and $m\times n$ zero matrix.

2. Preliminaries and problem formulation

2.1. Graph theory notions

Denote the directed graph by $\mathcal{G}=(\mathcal{V},\mathcal{E},A)$, where $\mathcal{V}$ and $\mathcal{E}$ represent, respectively, the node set and the edge set. Suppose that there exist n followers and one leader label as agents 1 to n, and agent r, respectively. Suppose $\mathcal{G}$ with order n be the interaction topology among the n followers. $A=[a_{ij}]\in\mathbb{R}^{n\times n}$ is the adjacency matrix associated with $\mathcal{G}$. Here $a_{ij}>0$ if agent i can obtain information from agent j and $a_{ij}=0$ otherwise. We assume that $a_{ii}=0$. Let $\overline{\mathcal{G}}=(\overline{\mathcal{V}},\overline{E},\overline{A})$ be a directed graph of order $n+1$ used to model the interaction topology among the n followers and one leader. The definitions of $\overline{\mathcal{V}}$, $\overline{\mathcal{E}}$, and $\overline{A}$ are similar to that of $\mathcal{V},\mathcal{E},A$. The (nonsymmetric) Laplacian matrix $L=[L_{ij}]\in\mathbb{R}^{n\times n}$ associated with A is defined as $l_{ii}=\sum_{j\neq i}a_{ij}$ and $l_{ij}=-a_{ij}$, where $i\neq j$. The diagonal matrix $B=diag\{b_1,\dots,b_n\}$ denotes the leader adjacency matrix associated with graph $\overline{\mathcal{G}}$, where $b_i>0$ if the ith follower can obtain the information from leader and $b_i=0$ otherwise. Here we assume that the leader does not receive information from the followers, which implies that $a_{rj}=0, j=1,\dots,n$.

In this paper, we suppose that the interaction topologies are Markovian switching. Let $\theta[k]$ be a homogeneous, finite-state, discrete-time Markov chain which takes values in a finite set $S\triangleq\{1,\dots,m\}$ with probability transition matrix Π. We assume that the Markov process is ergodic throughout this paper. Denote the switching topology set by $\widehat{\mathcal{G}}\triangleq\{\overline{\mathcal{G}}_1,\dots,\overline{\mathcal{G}}_m\}$.

2.2. Problem formulation

Suppose the dynamics of the ith follower is given by

$$\dot{\xi}_i(t)=u_i(t),\quad i=1,\dots,n \tag{1}$$

where $\xi_i(t)\in\mathbb{R}$ and $u_i(t)\in\mathbb{R}$ represent the position and the velocity, respectively. The leader's state considered is time-varying, which can be described as follows:

$$\begin{cases}\dot{\xi}_r(t)=v_r(t)\\ \dot{v}_r(t)=a(t)=a_r(t)+\delta(t)\end{cases} \tag{2}$$

where $\xi_r(t)\in\mathbb{R}$ and $v_r(t)\in\mathbb{R}, a(t)\in\mathbb{R}$ are, respectively, the position, the velocity and the acceleration of the leader. The assumptions of $a_r(t)$ and $\delta(t)$ are as the same as that in [17], that is, $a_r(t)$ is known and $\delta(t)$ is unknown but bounded with a given upper bound $\overline{\delta}$ (i.e. $|\delta(t)|\le\overline{\delta}$). It is assumed that only a subset of the followers can obtain the position of the leader, while cannot obtain the velocity and the acceleration of the leader. Hence, we employ the observer-type algorithm proposed in [17] as

$$u_i(t)=-\alpha\left[\sum_{j\in N_i(\theta(t))}a_{ij}^{\theta}(t)(\xi_i(t)-\xi_j(t))+b_i^{\theta}(t)(\xi_i(t)-\xi_r(t))\right]+v_i(t),\quad \alpha>0 \tag{3}$$

$$\dot{v}_i(t)=a_r(t)-\gamma\alpha\left[\sum_{j\in N_i(\theta(t))}a_{ij}^{\theta}(t)(\xi_i(t)-\xi_j(t))+b_i^{\theta}(t)(\xi_i(t)-\xi_r(t))\right],\quad \gamma>0, i=1,\dots,n. \tag{4}$$

Here, $v_i(t)$ in (3) is the estimated velocity of the leader obtained by the ith follower rather than the real velocity of the ith follower.

Using the direct discretization method in [18] to (2), we get that

$$\xi_r[k+1]=\xi_r[k]+Tv_r[k]+\frac{T^2}{2}a[k] \tag{5}$$

$$v_r[k+1]=v_r[k]+Ta[k] \tag{6}$$

where the definitions of $\xi_r[k]$, $v_r[k]$ and $a[k]$ are similar to that of continuous-time case, and T is the sampling period.

The discretized dynamics of (1) is

$$\xi_i[k+1]=\xi_i[k]+Tu_i[k]. \tag{7}$$

According to (3)–(7), the data-sampled algorithm for the ith follower is given as follows:

$$u_i[k]=-\alpha\left[\sum_{j\in N_i(\theta[k])}a_{ij}^{\theta}[k](\xi_i[k]-\xi_j[k])+b_i^{\theta[k]}(\xi_i[k]-\xi_r[k])\right]+\frac{T}{2}\hat{a}_i^{\theta[k]}[k]+v_i[k], \tag{8}$$

$$v_i[k+1]=v_i[k]+T\hat{a}_i^{\theta[k]}[k], \tag{9}$$

where $\xi_i[k]\in\mathbb{R}$ is the position of the ith follower at time $t=kT$, $\alpha>0$, $\gamma>0$ are the control gains to be determined, and

$$\hat{a}_i^{\theta[k]}[k]=a_r[k]-\alpha\gamma\left[\sum_{j\in N_i(\theta[k])}a_{ij}^{\theta[k]}(\xi_i[k]-\xi_j[k])+b_i^{\theta[k]}(\xi_i[k]-\xi_r[k])\right].$$

By denoting $\xi[k]\triangleq[\xi_1[k],\dots,\xi_n[k]]^T$, $v[k]\triangleq[v_1[k],\dots,v_n[k]]^T$, we can obtain the equation of the whole system as follows:

$$\xi[k+1]=\xi[k]-\alpha T(L^{\theta[k]}+B^{\theta[k]})\xi[k]+\alpha TB^{\theta[k]}\xi_r[k]\cdot\mathbf{1}_n+Tv[k]-\frac{T^2}{2}\alpha\gamma(L^{\theta[k]}+B^{\theta[k]})\xi[k]+\frac{T^2}{2}\alpha\gamma B^{\theta[k]}\xi_r[k]\cdot\mathbf{1}_n+\frac{T^2}{2}a_r[k]\cdot\mathbf{1}_n, \tag{10}$$

$$v[k+1]=v[k]-\alpha\gamma T(L^{\theta[k]}+B^{\theta[k]})\xi[k]+T\alpha_r[k]\cdot\mathbf{1}_n+\alpha\gamma TB^{\theta[k]}\xi_r[k]\cdot\mathbf{1}_n. \tag{11}$$

Also, by denoting $\overline{\xi}[k]\triangleq\xi[k]-\xi_r[k]\cdot\mathbf{1}_n$ and $\overline{v}[k]\triangleq v[k]-v_r[k]\cdot\mathbf{1}_n$, we obtain that the error dynamics of (10) and (11) are as follows:

$$\zeta[k+1]=C^{\theta[k]}\zeta[k]+W[k]\delta[k] \tag{12}$$

where

$$\zeta[k]=\begin{bmatrix}\overline{\xi}[k]\\ \overline{v}[k]\end{bmatrix},\quad C^{\theta[k]}=\begin{bmatrix}I_n-(\alpha T+\frac{1}{2}\alpha\gamma T^2)H^{\theta[k]} & TI_n\\ -\alpha\gamma TH^{\theta[k]} & I_n\end{bmatrix},$$

$$W=\begin{bmatrix}-\frac{T^2}{2}\cdot\mathbf{1}_n\\ -T\cdot\mathbf{1}_n\end{bmatrix},\quad H^{\theta[k]}=L^{\theta[k]}+B^{\theta[k]},$$

$L^{\theta[k]}$ is the (nonsymmetrical) Laplacian matrix associated with the adjacency matrix $A^{\theta[k]}$ and hence $\mathcal{G}^{\theta[k]}$, $B^{\theta[k]}$ is the leader adjacency matrix associated with graph $\overline{\mathcal{G}}^{\theta[k]}$. It follows from [20] that $\{\zeta[k], k\in\mathbb{N}\}$ is not a Markov process, but the joint process $\{\zeta[k],\theta[k]\}$ is. The initial state of the joint process is denoted by $\{\zeta_0,\theta_0\}$.

Remark 1. The algorithms in (3) and (4) can be found in [17], where the continuous-time tracking control problem was considered. In [8], the algorithms in (3) and (4) were discretized by data-sampled approach and the leader-following consensus problem was studied in the case where the topology is fixed. In this paper, we will extend the results in [8] to the case where the topologies