Neurocomputing 141 (2014) 223–235

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Enhanced distance regularization for re-initialization free level set evolution with application to image segmentation

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ARTICLE INFO

Article history:
Received 1 April 2013
Received in revised form
6 March 2014
Accepted 9 March 2014
Communicated by D. Tao
Available online 13 April 2014

Keywords:
Level set evolution
Re-initialization free method
Distance regularization
Forward and backward diffusion
Active contours
Image segmentation

ABSTRACT

In level set methods, the re-initialization remedy is widely applied to periodically replace the degraded level set function (LSF) with a signed distance function to maintain its regularity. Due to its various limitation, the energy functional regularization based methods using variational technique (e.g. distance, Gaussian, and reaction–diffusion based regularization methods) are recently introduced to replace this remedy. However, the relationship among them seems to be less investigated. In this paper, an enhanced distance regularized level set evolution (DRLSE-E) completely free of the re-initialization procedure is proposed based on analyzing these recent regularization models. DRLSE-E has an intrinsic capability of maintaining LSF's regularity, particularly the desirable signed distance property in a vicinity of the zero level set and the flat property out of this vicinity, which ensures accurate computation and stable level set evolution. Like other re-initialization free methods, DRLSE-E has simple and efficient numerical scheme in implementation, flexible initialization. Furthermore, DRLSE-E has the advantage of faster evolving speed and more numerical accuracy than distance regularized methods because of its forward and backward diffusion rate considering two competing components during the evolution. As an application example, DRLSE-E is used to typical edge-based and region-based active contour models for image segmentation and shows its competitiveness. Considering DRLSE-E is general, it can be easily incorporated into various existing level set models for image segmentation, filtering, and other tasks.

1. Introduction

In the last 20 years, active contour models (ACMs, snakes or deformable models) [1,2] have received extensive attention in the fields of image processing and computer vision, especially for image segmentation [3–5]. These methods all need to initialize a closed curve in the image and then evolve it until the evolving curve converges to the target [6,7]. According to the representation of the evolving curve, ACM can be roughly divided into parametric ACM [1,8] and geometric ACM [9]. The parametric ACM uses parametric equation to explicitly represent the evolving curve. The explicit representation easily brings some intrinsic drawbacks, such as difficulty in handling topological changes, limitation in capture range of concave boundaries, and dependency of parameterizations.

On the contrary, the geometric ACM implicitly represents the curves as the zero level set (LS) of a higher dimensional function, called LS function (LSF), and formulates the evolution of the curves through the evolution of LSF. The curve is evolved using the partial differential equation (PDE) derived from the energy function that describes a curve smoothing process. When the curve evolution stops, the zero LS corresponds to the segmentation result. The geometric ACM using the LS evolution (LSE) can naturally represent contour of complex topology and deal with topological changes (contour breaking and merging) without any extra functions by controlling the evolution of LSF rather than the parametric curves. Thus it significantly improves ACM by being free of the drawbacks in parametric ACM. Moreover, extensive numerical algorithms based on Hamilton–Jacobi equations have been developed, accurately handling shocks and providing stable numerical schemas [9,10]. These merits make LS methods a popular numerical technique for tracking moving interfaces or segmenting objects in image processing, computer vision, computer graphics, computational geometry, fluid mechanics, material sciences, etc. [11–15].

In LS evolution methods for image segmentation, the LSF is commonly defined by computing the closest distances between pixels and a given closed curve in an image domain. To obtain a clear zero LS as the boundary, the points that have positive

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http://dx.doi.org/10.1016/j.neucom.2014.03.011
0925-2312/

distances are inside the curve, and ones that have negative distances are outside the curve. Although the initial front would work for any function negative inside and positive outside, the signed distance function (SDF) gives many desirable properties [16]. Specifically, the SDF ϕ in a metric space $(\mathbf{X}, d)$ is defined by

$$\phi = \begin{cases} -d(\mathbf{x}, \partial S) & \text{if } \mathbf{x} \in S^- \\ 0 & \text{if } \mathbf{x} \in \partial S \\ d(\mathbf{x}, \partial S) & \text{if } \mathbf{x} \in S^+ \end{cases} \quad (1)$$

where $d(\mathbf{x}, S) = \inf_{\mathbf{y} \in S} d(\mathbf{x}, \mathbf{y})$ denotes the distance of a given point $\mathbf{x}$ to the boundary of S. If $S \subseteq \mathbb{R}^n$ with piecewise smooth boundary, the SDF is differentiable almost everywhere, and its gradient satisfies the Eikonal equation $|\nabla \phi| = 1$. This property close relates to the unit normal vector, the mean curvature, the closest point on the boundary, and the simplified volume and surface integrals on the domain. Therefore, the LSF is usually forced to be a SDF, especially in the evolving stage. However, during the iterative cycles of evolution, the size of the gradient of the SDF may become too small at certain points, that is, the property of SDF does not hold, which may cause numerical instabilities and even errors during computation. It is necessary to re-initialize the LS function as a SDF again to remedy this degeneracy or irregularities. In conventional LS methods, re-initialization is performed by periodically stopping the evolution and reshaping the degraded LSF as a SDF [17–19,10]. However, the use of re-initialization introduces some fundamental problems yet to be practically solved, such as no general answers to when and how to apply the re-initialization [20,21]. Re-initialization is often applied in an ad hoc manner, and it should be avoided as much as possible [18,22].

To reduce or eliminate the re-initialization step, the global minimization methods [23,24] are introduced to incorporate into some variational LSF via the specific form based total variation approach, such as Chan–Vese model [25] and Vese–Chan's piecewise smoothing model [26]. The convex object function in these methods is helpful for keeping the LSF's regularity during the evolution. From another aspect, the constrained diffusion-based LSE [27] is proposed to deal with the initialization dependency problem that commonly appears in edge-based approaches. The diffusion rate in this method changes smoothly from 0 to 1 and makes LSF tend to flat. By extending the radial basis functions (RBF) based LSE, a region based ACM not requiring any initialization [28,29] is further proposed. In this method, the LSF is interpolated using RBFs. Its shape and topology are determined by the coefficients of RBF interpolation. So, the finite difference based numerical methods to evolve the LSF are replaced by the adaptive changes of the RBF interpolation coefficients. The regularization of LSF is intrinsically handled through velocity normalization and the smoothing nature of RBF interpolation, and the periodic re-conditioning can be eliminated via RBF coefficient updating. Besides these methods, an efficient non-convex minimization algorithm [30] is proposed for distance preserving LS methods. This method overcomes the main numerical limitations by introducing constrained $\mathcal{L}_1$ optimization techniques via splitting this non-convex problem into sub-optimizations, and then combining them together using an augmented Lagrangian approach [31].

Recently, the variational LSE is introduced to eliminate the costly re-initialization procedure by incorporating a penalty term into the energy functional [32]. This method has received significant attentions because it repairs the critical but bottleneck-like disagreement between theory and implementation in LSE. Unfortunately, this penalty term may cause an undesirable side effect on the LSF in some circumstances, which may affect the numerical accuracy. To address this, an improved variational LS formulation [22] is further developed by incorporating a distance regularization term into the energy functional, and completely avoiding the undesirable side effect arisen from the penalty term. It is theoretically graceful and practically advisable for the investigation on a new double-well potential related to the distance regularization term to maintain a desired shape of the LSF.

Besides this, a Gaussian filter is recently proposed to regularize the LSF to achieve local segmentation and did not employ the re-initialization in numerical implementation [33]. In this method, the evolution of a function with its Laplacian derivation is equivalent to Gaussian filtering the initial condition of the function, so this method can be called as Gaussian regularized LS evolution (GRLSE). This regularization approach has received remarkable attentions because it is very fast, exact in segmenting clean objects, efficient in numerical implementation, and capable of adaptively selecting local or global segmentation. However, the energy functional in the penalty term is not explicitly presented, and the stability of the curves under the proposed LS formulation is not thoroughly investigated. More recently, a novel reaction–diffusion (RD) method [34] for implicit active contours is further presented, which is free of the costly re-initialization. The RD equation in phase transition modeling is based on the Van der Waals–Cahn–Hilliard theory in mechanics for stability analysis of systems with unstable components, and the RD term is introduced into penalty to derive a piecewise constant solution. Accordingly, this method performs well on weak boundary anti-leakage and noisy image. However, this method evolves slower than GRLSE.

We focus our topic on the regularization based variational LSE free of re-initialization. Although the four methods are advisable in image segmentation applications, there are some interesting questions among them. First, what is the intrinsic relationship among the distance regularized methods and the Gaussian regularized ones? Since they all are proposed independently in recent years, it seems that their relationship and their characteristic are less investigated. Second, in experiments it is found that the GRLSE method performs well for less iteration number but when the iteration number increases, it cannot segment the object anymore, what is reason for this? Third, the forward and backward diffusion is interesting in distance regularized methods, can the diffusion rate be further improved?

Motivated by these questions, in this paper, we proposed a new variational LSE, namely DRLSE-E, with an enhanced distance regularization energy term that drives the motion of the zero level contours toward desired locations. In our method, to maintain a desired shape of the LSF, particularly the desirable signed distance property in a vicinity of the zero LS and the flat property out at this vicinity, a new distance regularization term is defined to force the gradient magnitude of the LSF to one of its minimum points. It ensures accurate computation and stable LS evolution. The LSE is derived as a gradient flow that minimizes a double-well energy functional for the distance regularization term, which derives a forward-and-backward diffusion to maintain the regularity of LSF. Besides re-initialization-free property, the internal energy functional of DRLSE-E explicitly emphasizes the competition of signed distance preserving component and flat component for small gradient magnitude, while accelerates the gradient descent velocity for large gradient magnitude in different LSs, which makes DRLSE-E very simple and fast in numerical implementation. To validate DRLSE-E, we apply it to edge-based and region-based ACM for image segmentation, and compared them to some related methods. The experimental results on synthetic and real-world images demonstrate the advantage of DRLSE-E, e.g., implementation with a simpler and more efficient numerical scheme than conventional LSE, and relatively large time steps and computation time, while ensuring competitive numerical accuracy and efficiency.

The rest of this paper is organized as follows. In Section 2, we introduce some related works. In Section 3, we propose a new variational LSE with an enhanced distance regularization term.