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Output regulation of state-coupled linear multi-agent systems with globally reachable topologies

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ABSTRACT

This paper investigates output regulation problem of state-coupled linear certain and uncertain multi-agent systems with globally reachable topologies. Distributed dynamic state feedback control law is introduced to realize the regulator problem and a general global method for error regulation is established. The Jordan canonical form is used to stabilize the closed-loop control system. Sylvester equation and internal model theory are adopted to achieve the objectives of output regulation for every initial condition in the state space. Finally, numerical simulations are utilized to show the effectiveness of the obtained results.

1. Introduction

Recently, the coordination control for communication networks composed of multiple agents has received significant research attention in many fields. It is widely used on formation control, air traffic control, rendezvous, foraging, task and role assignment and cooperative search. Consensus of multi-agents means the agreement of a group of agents on their common states via the communication information based on the structural topology. Consensus algorithms have applications in vehicle formations [1,2], flocks [3,4], attitude alignment [5]. The whole systems can dispose complex tasks in a coordinated fashion. Multi-agent systems have more advantages than the conventional single control system on reducing cost, improving system efficiency, and producing new property and so on. In [6–12], the essential problem for multi-agent systems is to design a control law for each agent by using local information from other agents. Distributed consensus algorithms are designed, assuming only neighbor-to-neighbor interaction between agents.

In coordination control problems, the focus is on the communication constrains instead of the individual system dynamics [13,14]. The individual system dynamic is commonly modeled as simple integrator and the control input is based on the exchange of information modeled by some communication graph. In contrast to consensus problems, a particularly interesting topic called leader-following consensus problem is the consensus of a group of agents with a leader, where the leader is a special agent whose motion is independent of all the other agents and thus is followed by all the other ones [15–19]. A leader-following consensus problem of a group of autonomous agents with time-varying coupling delays was considered in [15]. The authors in [16] gave a leader-following consensus algorithm with communication input delays and then presented a frequency-domain approach to find the stability conditions. Distributed estimation via observers design for multi-agent leader-following was used in [18] where an active leader to be followed moved in an unknown velocity.

Output regulation is an important and interesting problem in control theory. This problem aims to achieve asymptotic tracking and disturbance rejection for a class of reference inputs and disturbances, which generated by an exosystem. Thus, the problem of output regulation is more challenging than stabilization and has attracted much attention. In multi-agent systems, exosystem is same for all the nodes but only partial nodes have the state information channel with it. The output regulation problem for linear or nonlinear systems had been studied, e.g., [20–25]. In recent years, output regulation of multi-agent systems had received considerable attention in [26–34]. It was shown in [26] that the partial control of the systems cannot access the exogenous signal. The robust output regulation problem of a multi-agent system was considered in [27], and internal model principle was used in an uncertain multi-agent system. Ref. [33] considered linear dynamical systems with heterogeneous networks. The adaptive regulator problem for linear systems had been addressed in [34].

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The objective of this paper is to research output regulation problem about state-coupled linear certain and uncertain multi-agent systems based on the relative states of neighboring agents and exosystem information. The reference inputs and/or the disturbances are same for all the nodes but only partial nodes have the state information of exosystem and the others cannot access the exogenous signal. In this case, a dynamic distributed compensator is established. A general global method for error regulation is established in this paper. The distributed dynamic state feedback control law based on compensator has been expressed under the globally reachable topologies. This paper is organized as follows: The system model and preliminaries are given in Section 2. The main results about certain and uncertain agents are presented in Sections 3 and 4. Following that, Section 5 gives numerical simulations, and finally, some conclusions are drawn in Section 6.

The following notations will be used throughout this paper. Let R be the set of real numbers. R^n is the n-dimensional vector space and $R^{n\times n}$ is the matrix space of dimension n. For a given vector or matrix A, $\sigma(A)$ denotes the spectrum of A. A^T denotes its transpose. $A\otimes B$ denotes the Kronecker product of matrices A and B. The Kronecker sum of $A\in R^{n\times n}$ and $B\in R^{m\times m}$ is defined as $A\oplus B=(A\otimes I_m)+(I_n\otimes B)$. 1_N represents $(1,1,\ldots,1)^T$ with dimension N.

2. Problem formulation and preliminaries

2.1. Algebraic graph theory

In this section, we review some preliminary graph theory in [35] which is a very useful mathematical tool in the research of multi-agent systems. The topology of a communication network can be expressed by a graph. Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ be a directed graph, where $\mathcal{V}$ is the set of nodes. $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges, and $\mathcal{A}=[a_{ij}]$ is a weighted adjacency matrix with nonnegative adjacency elements a_{ij}. The Laplacian with the directed graph is defined as $\mathcal{L}=\mathcal{D}-\mathcal{A}$, where $\mathcal{D}=[d_{ij}]$ is a diagonal matrix with $d_{ii}=\Sigma_{j=1}^{n}a_{ij}$. Obviously, all the row sums of $\mathcal{L}$ are zero. If the edge $e_{ij}=(v_i,v_j)\in\mathcal{E}$, then $a_{ij}>0$ which means agent i could receive information from agent j, other else $a_{ij}=0$. The set of neighbors of node v_i is denoted by $\mathcal{N}_i=\{v_j\in\mathcal{V}:(v_i,v_j)\in\mathcal{E}\}$. An edge of $\mathcal{G}$ denoted by $e_{ij}=(v_i,v_j)\in\mathcal{E}$ means that node v_i receives information from node v_j. There is a sequence of edges with the form (v_i,v_{k_1}), $(v_{k_1},v_{k_2}),\ldots,(v_{k_j},v_j)\in\mathcal{E}$ composing a direct path beginning with v_i ending with v_j, then node v_j is reachable from node v_i. A node is reachable from all the other nodes of graph, the node is called globally reachable.

2.2. System model

Suppose that the multi-agent systems under consideration consist of N agents. Directed graphs are used to model communication topologies. Each edge $(i,j)\in\mathcal{E}$ corresponds to a weighting information channel between agent i and j. The agent i is assumed to have the following dynamics:

$$\begin{aligned}\dot{x}_i(t)&=Ax_i(t)+Bu_i(t)+D_i(t)\\ y_i(t)&=Cx_i(t),\end{aligned}\tag{1}$$

where $x_i\in R^n$ is the state of ith subsystem. $u_i\in R^m$ is the consensus protocol to be designed which depends on the agent i and its neighbors. The term $D_i(t)$ represents a disturbance. $y_i\in R^p$ is the measurement output, i.e., the output can be used for the consensus protocol.

In addition, assume that there exists a finite dimensional linear system, representing the reference inputs and/or the disturbances, which is assumed to be generated by an exosystem

$$\begin{aligned}\dot{\omega}(t)&=\Gamma\omega(t)\\ D_i(t)&=E_i\omega(t),\end{aligned}\tag{2}$$

where $\omega\in R^q$ is the state of exosystem and E_i is a matrix with appropriate dimension which is associated with the description of disturbance signal, then

$$y_r(t)=Q\omega(t),\tag{3}$$

with $y_r(t)\in R^p$ as the reference output. The error output between the measurement output and reference output is represented as

$$e_i(t)=y_i(t)-y_r(t)=Cx_i(t)-Q\omega(t).\tag{4}$$

2.3. Problem statement

A digraph is used to describe the information communication between agents and the exosystem. Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A}_l)$ be a directed graph of order $N+1$, where $\mathcal{V}=\{0,1,2,\ldots,N\}$ is the set of nodes, in which the node indexed by 0 is referred to exosystem and the other nodes are corresponding to the agents be regulated. Edge set $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is often used to model the information exchange between agents, and $\mathcal{A}_l=[a_{ij}],i,j=0,1,2,\ldots,N$ is a weighted adjacency matrix of the digraph. The control u_i can receive the signal of the exogenous if and only if $a_{i0}>0$ and else $a_{i0}=0$. A digraph $\overline{\mathcal{G}}=(\overline{\mathcal{V}},\overline{\mathcal{E}},\overline{\mathcal{A}_l})$ which is used to label the agents except exosystem is defined as a subgraph of $\mathcal{G}$ with the vertex set $\overline{\mathcal{V}}=\{1,2,\ldots,N\}$. A dynamic compensator with the state $\zeta_i\in R^q, i=1,2,\ldots,N$, is established as

$$\dot{\zeta}_i(t)=\Gamma\zeta_i(t)+\alpha\left(\sum_{j\in\mathcal{N}_i}a_{ij}(\zeta_i(t)-\zeta_j(t))+a_{i0}(\zeta_i(t)-\omega(t))\right).\tag{5}$$

Note that the dynamics of ζ_i also depend on ζ_j, $j\in\mathcal{N}_i$, so (5) can always be seen as a distributed observer and the parameter α is an arbitrary constant which will be used later.

Let the external state measurements relative to its neighbors and the state-coupling variable relationship between agent i and $j\in\mathcal{N}_i$ be defined as

$$g_i(t)=\sum_{j\in N_i}a_{ij}(x_i(t)-x_j(t))+a_{i0}(x_i(t)-C^+Q\zeta_i(t)),\tag{6}$$

where a_{ij} is a weighted adjacency element of the digraph and C^+ is a generalized inverse of C.

To solve the output regulation problems, Distributed Dynamic State Feedback Control Law will be expressed in the form

$$\begin{aligned}u_i(t)&=K_1z_i(t)+K_2g_i(t)\\ \dot{z}_i(t)&=G_1z_i(t)+G_2\left(\sum_{j\in\mathcal{N}_i}a_{ij}(y_i(t)-y_j(t))+a_{i0}(y_i(t)-y_r(t))\right),\end{aligned}\tag{7}$$

with $z_i\in R^s$.

Remark 1. In this note, there exists the information exchange between agents and exosystem in the control law, but the agents have different dimensions with the exosystem. Eq. (6) is used to structure the external state measurements relative to its neighbors. As will be pointed out in Assumption (H3), it can also satisfy that C has full row rank and then we have $CC^+=I_p$.

Given the system (1), the error output (4) and the state feedback control law (7), let

$$\begin{aligned}x&=[x_1^T,\ldots,x_N^T]^T,\quad z=[z_1^T,\ldots,z_N^T]^T\\ \zeta&=[\zeta_1^T,\ldots,\zeta_N^T]^T,\quad e=[e_1^T,\ldots,e_N^T]^T\end{aligned}$$

and $\tilde{\omega}=\mathbf{1}_N\otimes\omega$. We can obtain the system as follows:

$$\begin{aligned}\dot{x}(t)=&(I_N\otimes A+H\otimes BK_2)x(t)+(I_N\otimes BK_1)z(t)\\ &-(A_0\otimes BK_2C^+Q)\zeta(t)+E\tilde{\omega}(t)\end{aligned}$$