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Projective synchronization of different chaotic neural networks with mixed time delays based on an integral sliding mode controller

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A B S T R A C T

In this paper, an integral sliding mode control (SMC) approach is presented to investigate the projective synchronization of nonidentical chaotic neural networks with mixed time delays. By considering a proper sliding surface and constructing Lyapunov–Krasovskii functional, as well as using the linear matrix inequality (LMI) technique, a sliding mode controller is designed to achieve the projective synchronization of the different neural networks. Finally, numerical simulations are carried out to illustrate the effectiveness of the main results.

1. Introduction

Over the past few years, neural networks have attracted much attention due to the background of a wide range of application such as associative memory, pattern recognition, image processing and model identification [1–6]. As we know, there exist time delays in the information processing of neurons due to various reasons. The existence of time delays may lead to some complex dynamic behaviors such as oscillation, divergence, chaos, instability, or other poor performance of the neural networks [3–11].

Since Pecora and Carroll introduced a method to realize the synchronization of two identical chaotic systems with different initial conditions [12], the synchronization of chaotic systems has attracted considerable attention [13]. There are several types of synchronization which have been found in interesting chaotic systems, such as antiphase synchronization [14], generalized synchronization [15], anticipatory synchronization [16], lag synchronization [17]. A projective synchronization phenomenon was first reported and discovered by Gonzalez-Miranda [18]. In 1999, Mainieri and Rehacek declared that the two identical systems could be synchronized up to a scaling factor α [19]. Subsequently, some researchers extended the concept of projective synchronization and termed it as generalized projective synchronization [20–22].

Most of the previous works (see, e.g., [14–16,23–28]) on the projective synchronization of chaotic delayed neural networks always assume that the drive and the response systems have identical dynamic structure [14–16] and the same parameters [25,26]. But in many practical situations, the drive and the response systems are different. Therefore, it is interesting to study different neural networks with mixed time delays both in theory and in applications.

Motivated by the above discussions, this paper investigates the projective synchronization problem of different chaotic neural networks with both the discrete and distributed time delays. To overcome the difficulty that complete synchronization between nonidentical chaotic neural networks cannot be achieved only by utilizing output feedback control, the sliding mode control approach is presented. To guarantee that the response system can be projectively synchronized with the drive system, the integral sliding mode control approach is presented, as well as Lyapunov–Krasovskii functional and the linear matrix inequality are used. Furthermore, numerical simulations are carried out to illustrate the effectiveness of the main results.

Throughout this paper, R^n and $R^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. The superscript T denotes matrix transposition and I denotes the identity matrix. $P>0$ means that is a real symmetric positive definite matrix. $*$ represents the elements below the main diagonal of a symmetric matrix.

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2. Problem description

In this paper, we consider the delayed neural network model defined by the following equations:

$$\dot{x}(t)=-C_1x(t)+A_1f_1(x(t))+B_1f_2(x(t-\tau_1))+D_1\int_{t-\tau_2}^{t}f_3(x(s))\,ds+J_1 \tag{1}$$

where $x_i(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T\in R^n$ is an n-dimensional state vector of the neural networks; $C_1=diag(c_1^1,c_2^1,\ldots,c_n^1)$ is the state feedback coefficient matrix; $A_1=(a_{ij}^1)_{n\times n}$, $B_1=(b_{ij}^1)_{n\times n}$ and $D_1=(d_{ij}^1)_{n\times n}$ are, respectively, the connection weight matrix, the discretely delayed connection weight matrix and distributive delayed connection weight matrix. J_1 is an external input vector; $\tau_i\geq 0\ (i=1,2)$ is a transmission delay; $f_i(x(t))=(f_{i1}x(t), f_{i2}(x(t)),\ldots,f_{in}(x(t))^T\ (i=1,2,3)$ denote the neuron activation functions. The initial conditions of (1) are given by $x_i(t)=\mu_i(t)\in C([-\tau_{\max},0],R)$, where $C([-\tau_{\max},0],R)$ denotes the set of all continuous functions from $[-\tau_{\max},0]$ to R. Here $\tau_{\max}=\max\{\tau_1,\tau_2\}$.

Let the neural network (1) be the drive system. The response system is given as

$$\dot{y}(t)=-C_2y(t)+A_2g_1(y(t))+B_2g_2(y(t-\tau_1))+D_2\int_{t-\tau_2}^{t}g_3(y(s))\,ds+u(t)+J_2, \tag{2}$$

where $y(t)\in R^n$ is the state vector of the response system, $g_i(x(t))\ (i=1,2,3)$ denotes the neuron activation function and $u(t)$ is the control input to be designed.

Throughout this paper, we assume that there exist positive constants $F_{1i},F_{2i},F_{3i}>0,\ i=1,2,\ldots,n$ such that the activation function f_i satisfies the following conditions:

$$0\leq\frac{f_{1i}(u_i)-f_{1i}(v_i)}{u_i-v_i}\leq F_{1i},\quad 0\leq\frac{f_{2i}(u_i)-f_{2i}(v_i)}{u_i-v_i}\leq F_{2i},\quad 0\leq\frac{f_{3i}(u_i)-f_{3i}(v_i)}{u_i-v_i}\leq F_{3i} \tag{3}$$

where $u_i,v_i\in R$.

To prove the main result, Lemma 1 is presented.

Lemma 1 (*Gu et al. [29]*). *For any positive definite matrix $D\in R^{n\times n}$, a scalar $\rho>0$, vector function $\omega:[0,\rho]\to R^n$ such that the integration concerned is well defined, then*

$$\left(\int_0^{\rho}\omega(x)\,ds\right)^T D\left(\int_0^{\rho}\omega(x)\,ds\right)\leq\rho\int_0^{\rho}\omega(x)D\omega(x)\,ds.$$

Definition 1. The projective synchronization error between systems (1) and (2) is defined as $e(t)=y(t)-\alpha x(t)$, where $\alpha\neq 0$ is called a scaling factor.

The objective of this study is to propose an approach to design a suitable controller $u(t)$ such that

$$\lim_{t\to\infty}\|e(t)\|=\lim_{t\to\infty}\|y(t)-\alpha x(t)\|=0, \tag{4}$$

where $\|\cdot\|$ denotes the Euclidean norm of a vector. If Eq. (4) is satisfied, then we can say that systems (1) and (2) have obtained projective synchronization. According to the Definition 1, the error system can be obtained from (1) and (2) as follows:

$$\begin{aligned}\dot{e}(t)=&-C_2e(t)+A_2\Phi_1(e(t))+B_2\Phi_2(e(t-\tau_1))\\&+D_2\int_{t-\tau_2}^{t}\Phi_3(e(s))\,ds+u(t)+(J_2-\alpha J_1)\\&+\alpha[A_2g_1(x(t))-A_1f_1(x(t))]+\alpha(C_1-C_2)x(t)\\&+\alpha[B_2g_2(x(t-\tau_1))-B_1f_2(x(t-\tau_1))]\\&+\alpha\left[D_2\int_{t-\tau_2}^{t}g_3(x(s))\,ds-D_1\int_{t-\tau_2}^{t}f_3(x(s))\,ds\right]\end{aligned} \tag{5}$$

where $\Phi_i(e(t))=g_i(y(t))-\alpha g_i(x(t)),\ i=1,2,3.$

3. Projective synchronization based on sliding mode control

3.1. Sliding surface and equivalent control law design

It can be seen clearly from (5) that the dynamic behavior of the error system depends on both error state $e(t)$ and chaotic state $x(t)$ of the drive system (1). Therefore, projective synchronization between different chaotic neural networks (1) and (2) cannot be achieved only by utilizing output feedback control. To overcome the difficulty, an integral sliding mode control approach will be proposed to overcome this difficulty and realize it.

Sliding mode control, as an effective robust control strategy, has been successfully applied to a wide variety of complex systems and engineering, including uncertain systems, time-delay systems, stochastic systems, singular systems and Markovian jump systems [30–32]. Using the sliding mode control method to synchronize drive-response chaotic systems involves two major stages: (1) selection of an appropriate switching surface for the desired sliding motion; (2) design of a sliding mode control law that brings any orbit in phase space to the switching surface and then achieves the synchronization of drive-response chaotic systems even in the presence of parameter and disturbance uncertainties.

For the sliding mode controller design in this paper, an appropriate switching surface with integral operation is designed such that the sliding motion on the manifold has the desired properties. The sliding mode controller is constructed as

$$\begin{aligned}S(t)=&e(t)+\int_0^t K[z(s)-\alpha K_1x(s)-\alpha K_2x(s-\tau_1)-\alpha K_3x(s-\tau_2)]\,ds\\&-\int_0^t[-C_2e(s)+A_2\Phi_1(e(s))+B_2\Phi_2(e(s-\tau_1))]\,ds\\&-D_2\int_0^t\int_{s-\tau_2}^{s}\Phi_3(e(\xi))\,d\xi\,ds\end{aligned} \tag{6}$$

where $K\in R^{n\times n}$ is a gain matrix to be designed; $K_1,K_2,K_3\in R^{n\times n}$ are known constant matrices, $z(t)=K_1y(t)+K_2y(t-\tau_1)+K_3y(t-\tau_2)$.

It follows from (5) and (6) that

$$\begin{aligned}S(t)=&e(0)+\int_0^t\{KK_1e(s)+KK_2e(s-\tau_1)+KK_3e(s-\tau_2)+\alpha(C_1-C_2)x(s)\\&+(J_2-\alpha J_1)+u(t)+\alpha[A_2g_1(x(s))-A_1f_1(x(s))+B_2g_2(x(s-\tau_1))\\&-B_1f_2(x(s-\tau_1))]\}\,ds+\alpha D_2\int_0^t\int_{s-\tau_2}^{s}g_3(x(\xi))\,d\xi\,ds\\&-\alpha D_1\int_0^t\int_{s-\tau_2}^{s}f_3(x(\xi))\,d\xi\,ds\end{aligned} \tag{7}$$

According to the sliding mode control theory, it is true that $S(t)=0$ and $\dot{S}(t)=0$ as the state trajectory of the error system (5) enters into the sliding mode. It thus follows from (7) and $\dot{S}(t)=0$ that an equivalent control law can be designed as

$$\begin{aligned}u_{eq}(t)=&-KK_1e(t)-KK_2e(t-\tau_1)-KK_3e(t-\tau_2)-(J_2-\alpha J_1)\\&-\alpha(C_1-C_2)x(t)-\alpha[A_2g_1(x(t))-A_1f_1(x(t))+B_2g_2(x(t-\tau_1))\\&-B_1f_2(x(t-\tau_1))]-\alpha D_2\int_{t-\tau_2}^{t}g_3(x(s))\,ds+\alpha D_1\int_{t-\tau_2}^{t}f_3(x(s))\,ds.\end{aligned} \tag{8}$$

Substituting (8) into (5), the sliding mode dynamics can be obtained and described by

$$\dot{e}(t)=-(C_2+KK_1)e(t)-KK_2e(t-\tau_1)-KK_3e(t-\tau_2)+A_2\Phi_1(e(t))$$