Neurocomputing 175 (2016) 188–197

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Local and global regularized sparse coding for data representation

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ARTICLE INFO

Article history:
Received 26 May 2015
Received in revised form
26 September 2015
Accepted 22 October 2015
Communicated by Yue Gao
Available online 2 November 2015

Keywords:
Sparse coding
Data representation
Regularizer
Regression
Clustering

ABSTRACT

Recently, sparse coding has been widely adopted for data representation in real-world applications. In order to consider the geometric structure of data, we propose a novel method, *local and global regularized sparse coding (LGSC)*, for data representation. LGSC not only models the global geometric structure by a global regression regularizer, but also takes into account the manifold structure using a local regression regularizer. Compared with traditional sparse coding methods, the proposed method can preserve both global and local geometric structures of the original high-dimensional data in a new representation space. Experimental results on benchmark datasets show that the proposed method can improve the performance of clustering.

1. Introduction

Over the past decade, data representation has attracted increasing attention in computer vision, information retrieval and machine learning. In many applications [1–4], processing high dimensional data in classification or clustering tasks is still a big challenge. To improve the performance of classification or clustering, a common way is to seek a meaningful low dimensional representation of the high dimensional data by dimensionality reduction or matrix factorization approaches.

Principal component analysis (PCA) [5] and linear discriminant analysis (LDA) [6] are two widely used linear representation methods. The former is an unsupervised learning approach, which aims to preserve the global covariance structure of data. The latter is a supervised learning method, which extracts the optimal discriminant vectors when class labels of data are available. However, both of them can not discover the latent manifold structure among data. In the past a few years, many methods have been proposed to address this issue. Among them, the most representative methods are ISOMAP [7], locally linear embedding (LLE) [8] and Laplacian Eigenmaps (LE) [9]. Although these manifold learning methods have achieved impressive results on data visualization, they cannot deal with the 'out-of-sample' problem. He et al. [10] proposed a linear version of the LE, namely locality preserving projection (LLP), which can alleviate this drawback. Several data representation methods, such as local and global regressive mapping (LGRM) [11], graph regularized nonnegative matrix factorization (GNMF) [12], local learning regularized nonnegative matrix factorization (LLRNMF) [13], locally consistent concept factorization (LCCF) [14] and local regularized concept factorization (LCF) [15], have been developed to exploit the geometric manifold structure of data. Extensive experimental results have demonstrated the effectiveness of these techniques.

In recent years, sparse coding (SC) has shown great success in data representation and a range of applications such as image processing [16–18], classification [19–22], and visual analysis [23–26]. Essentially, SC seeks to linearly represent a test sample by only a few training samples, which leads to the sparsity of the representation coefficient. To achieve sparse representation, many methods have been developed in the past few years, e.g. sparse PCA [27], sparse NMF [28], and sparse low-rank representation [29]. However, in conventional sparse coding methods, a common drawback is that some prior knowledge of data has been neglected, such as the geometric structure information. Wang et al [30] presented a novel sparse coding method, called locality-constrained linear coding (LLC). Furthermore, in order to preserve the spatial consistency, locally-invariant sparse representation were proposed by pooling the sparse coefficients across overlapping windows [31]. Mairal et al. [32] introduced a simultaneous sparse coding method by jointly decomposing groups of similar signals on subsets of the learned dictionary, which was

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http://dx.doi.org/10.1016/j.neucom.2015.10.048
0925-2312/

implemented using a group-sparsity regularizer. Zheng et al. [33] proposed a graph regularized sparse coding (GSC) method for image representation. In GSC, the geometric manifold structure of data is taken into account by imposing the graph regularizer. Thus, GSC performs significantly better than the traditional sparse coding methods on several benchmark databases. However, GSC only utilizes the local manifold structure of data by the regularization technique, and neglects the global geometric relationship of data. Therefore, a better approach is expected to learn a lower-dimensional representation to preserve both local and global structure of data, which is beneficial for achieving promising performance.

Motivated by the recent progresses in sparse coding and manifold learning, in this paper, a novel method, *local and global regularized sparse coding (LGSC)*, is proposed to represent the high dimensional data. Compared with traditional sparse coding methods, the proposed LGSC not only considers the manifold structure of data by constructing a local regression predictor, but also preserves its global structure using a global regression regularizer. Experimental results on several bench mark datasets have validated the proposed the effectiveness of the LGSC methods.

It is worthwhile to highlight the main contributions of this work as follows:

(1) We employ the local regression to model the local manifold structure, and simultaneously use the global regression as a regularization term to capture the global structure of data. In LGSC, both local and global regression regularization terms are combined into an integrated regularizer, which captures the intrinsic geometric structure of real-world data.
(2) In LGSC, the integrated regularizer is incorporated into the traditional sparse coding method, which makes LGSC more discriminative. In addition, we develop an iterative update scheme to solve the optimization problem of the LGSC and present the convergence curve in this paper.
(3) We conduct comprehensive experiments to analyse and compare our method with several state-of-the-art methods. The experimental results on real world image datasets demonstrate that the proposed method is superior to other data representation methods.

The rest of this paper is organized as follows. The sparse coding and GSC methods are reviewed in Sections 2.1 and 2.2. The proposed LGSC method is described in Section 3. Experimental results are presented in Section 4. Finally, conclusions are drawn in Section 5.

2. Related works

This section contains description of related works to the proposed approach, i.e. SC and GSC.

2.1. Sparse coding

Sparse coding aims to linearly represent a sample by a few atoms in a dictionary. Given a data set $X=[x_1,x_2,\dots,x_n]\in R^{m\times n}$ with n data points sampled from an m-dimensional feature space. Let $D\in R^{m\times k}$ be an over-complete dictionary and $A\in R^{k\times n}$ be the representation coefficient, where k denotes the number of the atoms. In order to achieve the sparsity of coding coefficients, the l_0-norm is used to constrain the representation coefficient. Mathematically, the minimization problem of sparse coding can be formulated as

$$\min_{D,A}\|X-DA\|_F^2+\alpha\sum_{i=1}^{m}\|a_i\|_0$$
$$\text{s.t.}\|d_i\|^2\le c,\ i=1,\dots,k \tag{1}$$

where $\|\cdot\|_F$and $\|\cdot\|_0$ denote the Frobenius norm of a matrix and the l_0-norm of a vector, respectively, c is a given constant and α is a constant parameter. Solving the l_0-norm minimization problem is NP-hard. Fortunately, it can be replaced by an l_1-norm minimization problem if the solution of Eq. (1) is sufficiently sparse [34,35]. Therefore, the optimization problem in Eq. (1) can be rewritten as follows:

$$\min_{D,A}\|X-DA\|_F^2+\alpha\sum_{i=1}^{m}\|a_i\|_1$$
$$\text{s.t.}\|d_i\|^2\le c,\ i=1,\dots,k \tag{2}$$

where $\|\cdot\|_1$ denotes the l_1-norm of a vector. Since the l_1-norm minimization problem in Eq. (2) is a convex optimization problem, it can be efficiently solved using existing software packages such as l_1-magic [36], PDCO-LSQR [37] and PDCO-CHOL[37].

2.2. Graph regularized sparse coding

Previous studies [7–9] have shown that manifold learning plays an important role in data representation. A natural assumption is that if two data samples are close in the original feature space, then their low dimensional representation should be close to each other in the new representation space. This is usually referred to as the *manifold learning assumption*. Using graph regularization techniques, GSC can discover the latent manifold structure of data.

Given a set of data points $X=[x_1,x_2,\dots,x_n]\in R^{m\times n}$, the geometric structure of data can be characterized by a k-nearest neighbor graph $G=\{X,W\}$with a vertex set X and an affinity weight matrix W. If x_i is among the k-nearest neighbors of x_j or x_i is among the k-nearest neighbors of x_i, $W_{ij}=1$, otherwise, $W_{ij}=0$. The graph regularization term is expressed as follows:

$$\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}(a_i-a_j)W_{ij}=Tr(ALA^T) \tag{3}$$

where $A=[a_1,\cdots,a_n]$ is the sparse coefficient matrix, $L=D-W$ is the Laplacian matrix, D is a diagonal matrix and $D_{ii}=\sum_j W_{ij}$.

By incorporating the Laplacian regularizer (3) into sparse coding, the objective function of GSC can be expressed as follows:

$$\min_{D,A}\|X-SA\|_F^2+\alpha Tr(ALA^T)+\beta\sum_{i=1}^{m}\|a_i\|_1$$
$$\text{s.t.}\|a_i\|^2\le c,\ i=1,\dots,k \tag{4}$$

where α and β are the regularization parameters. The optimization problem in Eq. (4) can be solved by the feature search algorithm proposed in [38].

3. The proposed method

We start this section by discussing the motivation of our work. Then we introduce the proposed LGSC method in detail.

3.1. Motivation

Sparse coding is a typical data representation method based on an over-complete dictionary. Most of sparse coding methods, however, fail to make full use of the geometrical structure of data. In fact, the intrinsic structure of data is unknown and complex in many real-world applications. Thus, a single global or local graph may be insufficient to characterize the underlying geometrical structure of data. A reasonable approach should integrate both local and global structures of data in the representation step.