Neurocomputing 175 (2016) 278–286

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Dynamical group consensus of heterogenous multi-agent systems with input time delays

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ARTICLE INFO

Article history:
Received 7 July 2015
Received in revised form
19 August 2015
Accepted 20 October 2015
Communicated by Liang Wang
Available online 30 October 2015

Keywords:
Group consensus
Heterogenous multi-agent systems
Cooperative control
Time delay
Graph theory

ABSTRACT

This paper investigates the dynamics group consensus problem of heterogenous multi-agent systems with time delays, in which agents' dynamics are modeled by single integrators and double integrators. To achieve group consensus, a class of dynamics group consensus protocols is proposed for heterogenous multi-agent systems with input time delays, which can also be used to solve group consensus for heterogenous multi-agent systems without input time delays. By using frequency-domain analysis method and matrix theory, some sufficient group consensus conditions, which are dependent on the input delays and the control parameters, are obtained for heterogenous multi-agent systems under directed and undirected communication topologies with and without input time delays, respectively. Simulation results are also provided to illustrate the effectiveness of the obtained results.

1. Introduction

Cooperative control of multi-agent systems has been extensively studied in last decade. Consensus problems, as a branch of cooperative control, has received considerable attention recently in many fields, such as biology, physics, robotics and control engineering [1–6]. Consensus of multi-agent systems means to design some appropriate protocols and controllable strategies via local interaction such that all the agents reach an agreement on their common interest quantity. The interest quantity might represent attitude, position, velocity, temperature, voltage, and so on. In the literature, there are many interesting research results on consensus problems for multi-agent systems, whose agents with first-order dynamics [7–11], second-order dynamics [12–14], high-order dynamics [15–17] and general linear dynamics [18].

Note that almost all papers concern the complete consensus that the states of all agents converge to a common consistent state. However, in practice a phenomena frequently occurs that agreements are different with the change of environments, situations, cooperative tasks or even time. Consequently, a critical problem is to design appropriate protocols such that agents in systems can reach more than on consistent states. This is called group (or cluster) consensus problem in multi-agent systems, in which the agents in a system are divided into multiple subgroups and different subgroups can reach different consistent states. Recently, great deals of excellent research results on group consensus have emerged constantly. In Ref. [19] Yu et al. solved the group consensus problem for multi-agent systems with fixed undirected topology. Sequently, in Ref. [20] Yu et al. extended their results in Ref. [19] to multi-agent systems with switching topologies and communication delays by using the method of double-tree-form. In [21], the authors proposed a hybrid protocol to solve the couple-group average-consensus problem of multi-agent systems with a fixed topology. In [22], Feng et al. studied the static and time-varying group consensus problems of second-order multi-agent systems. In [23], Liao et al. investigated group consensus of dynamical multi-agent networks via pinning scheme.

Most of aforementioned works on consensus problems are on the homogenous multi-agent systems, in which all the agents have the same dynamics. In practical applications, the dynamics of agents may be different because of common goals of mixed agents or various restrictions of communication costs [24]. Recently, more and more attention has been paid to the consensus problems for heterogenous multi-agent systems, where agents have different dynamics. In [25], Zheng and Wang studied the consensus problem of the heterogeneous multi-agent systems consisted of first-order integrator agents and second-order integrator agents, in which a linear consensus protocol and a saturated consensus protocol were proposed. In [26], Zhu et al. studied the finite time consensus problems for heterogeneous multi-agent systems. In [27], the consensus problem of heterogeneous multi-agent systems consisted of first-order agents,

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http://dx.doi.org/10.1016/j.neucom.2015.10.060
0925-2312/

second-order agents and Euler–Lagrange agents. In [28], the consensus seeking problem of heterogeneous multi-agent systems consisted of first-order agents and second-order agents with input delays was investigated. In [29], Liu et. al investigated second-order consensus of multi-agent systems with heterogeneous nonlinear dynamics and time-varying delays by introducing novel decentralized adaptive strategies to both the coupling strengths and the feedback gains.

Here, it is worth noting that almost all of the aforementioned works concern consensus/group consensus problems for homogeneous multi-agent systems or consensus problems for heterogeneous multi-agent systems. However, there are few results on the group consensus of heterogeneous multi-agent systems consisting of first-order and second-order agents. Moreover, to the best of our knowledge, there is no paper concerning group consensus of heterogeneous multi-agent systems with time delays. Motivated by the above discussion, this paper investigates the dynamics group consensus problem of heterogenous multi-agent systems with time delays. The main contributions of this paper can be stated as follows: A class of distributed group consensus protocols is proposed for heterogenous multi-agent systems with input time delays, which also can be used to solve group consensus for heterogenous multi-agent systems without input time delays. By using frequency-domain analysis method and matrix theory, some sufficient group consensus conditions, which are dependent on the input delays and the control parameters, are obtained for heterogenous multi-agent systems under directed and undirected communication topologies with and without time delays, respectively. Simulation results are also provided to illustrate the effectiveness of the obtained results. Compared with the existing references, this paper has the following advantages: Firstly, in contrast of consensus/group consensus for homogeneous multi-agent systems [10–15,17,18,20,19,21–23], we investigate the group consensus problems for heterogenous multi-agent systems. Secondly, in contrast to the existing results in [28,25,26,29], where consensus problems were considered for heterogenous multi-agent systems only under undirected communication topologies, in this paper we consider the group consensus underdirected and undirected communication topologies. Thirdly, in contrast to the existing results in [23–26], where consensus/group consensus problems were investigated without considering time delays, in this paper, group consensus problems are investigated with and without considering time delays, respectively.

The rest of this paper is organized as follows. In Section 2, some preliminaries on graph theory are presented. In Section 3, the problem description is given. In Section 4, the main results on dynamics group consensus problem of heterogenous multi-agent systems with time delays are presented. In Section 5, numerical examples are simulated. Conclusions are finally drawn in Section 6.

2. Preliminaries

2.1. Algebraic graph theory

Algebraic graph theory is a natural framework to study cooperative control problems of multi-agent systems. Using algebraic graph theory, the information exchanged among agents can be modeled by an interaction graph. Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ be a weighted directed or undirected graph of order n with the finite nonempty set of nodes $\mathcal{V}=\{v_1,\dots,v_n\}$, the set of edges $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $\mathcal{A}=(a_{ij})_{n\times n}$. Here, each node v_i in $\mathcal{V}$ corresponds to an agent i, and each edge $(v_i,v_j)\in\mathcal{E}$ in a weighted directed graph corresponds to an information link from agent j to agent i, which means that agent i can receive information from agent j. In contrast, the pairs of nodes in weighted undirected graph are unordered, where an edge $(v_j,v_i)\in\mathcal{E}$ denotes that agent i and j can receive information from each other. The weighted adjacency matrix $\mathcal{A}$ of a weighted directed graph is defined such that $a_{ii}=0$ for all $v_i\in\mathcal{V}, a_{ij}\neq 0$ if $(v_j,v_i)\in\mathcal{E}$, and $a_{ij}=0$ otherwise. The weighted adjacency matrix $\mathcal{A}$ of a weighted undirected graph is defined analogously except that $a_{ij}=a_{ji}, \forall i\neq j$, since $(v_i,v_j)\in\mathcal{E}$ implies $(v_j,v_i)\in\mathcal{E}$. We can say v_i is a neighbor vertex of v_j, if $(v_i,v_j)\in\mathcal{E}$. A directed path is a sequence of ordered edges of the form (v_{i_1},v_{i_2}), $(v_{i_2},v_{i_3}),\dots$, where $(v_{i_j},v_{i_{j+1}})\in\mathcal{E}$. If there is a path in $\mathcal{G}$ from one node v_i to another node v_j, then v_j is said to be reachable from v_i. If not, then v_j is said to be not reachable from v_i. If there is a node that is reachable from every other node in the digraph, then we say the node is global reachable.

$L=(l_{ij})_{n\times n}$ is the Laplacian matrix of graph $\mathcal{G}$, and is defined by $l_{ij}=-a_{ij}$ for $i\neq j$, and $l_{ii}=\sum_{j=1,j\neq i}^{n}a_{ij}$, $i,j\in\{1,\dots,n\}$. For an undirected graph, L is symmetric positive semi-definite. However, L is not necessarily symmetric for a directed graph.

Lemma 2.1 (*Lin et al. [30]*). *0 is a simple eigenvalue of the Laplacian matrix L, and $\mathbf{1}_n=[1,\dots,1]^T$ is the corresponding right eigenvector, that is, $L\mathbf{1}_n=0$, if and only if the digraph $G=(\mathcal{V},\mathcal{E},\mathcal{A})$ has a globally reachable node.*

2.2. Notations

We use standard notations throughout this paper. I_n represents the $n\times n$ identity matrix, $0_{m\times n}$ represents the $m\times n$ zero matrix, and $\mathbf{1}_n=[1,1,\dots,1]^T\in R^n$ ($\mathbf{1}$ for short, when there is no confusion). $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are the smallest and the largest eigenvalues of the matrix A respectively. $\rho(\cdot)$, det $(\cdot)$ represent the spectral radius, determinant of a matrix, respectively. R, C and Z_+ denote the sets of real numbers, complex numbers and positive integers. For $s\in C$, $|s|$, Re(s) and Im(s) denote its modulus, real and imaginary part, respectively. $arg(\cdot)$ denotes the phase of a complex number.

3. Problem description

In this paper, we consider the group consensus problems under input time delays for heterogeneous multi-agent systems composed of double integrator agents and single integrator agents. Without loss of generality, we suppose there are m double integrator agents described by

$$\begin{aligned}\dot{x}_i(t)&=v_i(t)\\ \dot{v}_i(t)&=u_i(t-\tau),\quad i=1,\dots,m,\end{aligned}\tag{1}$$

and $n-m$ single integrator agents described by

$$\dot{x}_i(t)=u_i(t-\tau),\quad i=m+1,\dots,n,\tag{2}$$

where $x_i\in R^p$, $v_i\in R^p$ and $u_i\in R^p$ are the position, velocity and control input of agent i, respectively. For the simplicity of analysis, in the following, we only consider the case where $p=1$. The analysis and main results still hold for any dimension p by using Kronecker product.

Group consensus requires that the first m agents reach one consistent state while the last $n-m$ agents reach another consistent state in the presence of information exchange between the two groups. Denote $\ell_1=\{1,2,\dots,m\}$, $\ell_2=\{m+1,m+2,\dots,n\}$.

Definition 1. The heterogeneous multi-agent system (1) and (2) is said to be reach group consensus asymptotically if for any initial state values $x_i(0)$ and $v_i(0)$, we have

$$\lim_{t\to\infty}|x_i(t)-x_j(t)|=0,\ \lim_{t\to\infty}|v_i(t)-v_j(t)|=0,\ \forall i,j\in\ell_k,\quad k=1,2.\tag{3}$$

Now the following necessary assumptions are made.