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Fault diagnosis and fault tolerant tracking control for the non-Gaussian singular time-delayed stochastic distribution system with PDF approximation error

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ABSTRACT

In this paper, fault diagnosis and fault tolerant tracking control algorithms are proposed for the non-Gaussian singular time-delayed stochastic distribution control system. The square root B-spline model is used to approximate the output probability density function (PDF), and the PDF approximation error is taken into consideration. A fault diagnosis approach based on the adaptive observer is constructed to diagnose the size of fault in the singular stochastic distribution control (SDC) system. When fault occurs, in order to track the expected PDF, a distribution tracking error dynamic system is established. The purpose of fault tolerant tracking control is transformed into making the distribution tracking error at each time instant satisfy a certain upper bound beyond a limited time. Then the fault tolerant controller is designed using the fault diagnosis information and other measurable information. A simulation example is included to illustrate the effectiveness of the proposed algorithms and encouraging results have been obtained.

1. Introduction

Stochastic distribution control (SDC) is a new branch of stochastic system control in which the system output is the non-Gaussian probability density function (PDF) of the system output. SDC theory was proposed by Professor Wang Hong in 1996 [1], which has drawn considerable attentions in the past decades. Many interesting control algorithms such as robust tracking control, fault tolerant control and minimum entropy control algorithms have been developed [2–5]. In literature 5, a robust tracking controller via an augmentation control and linear matrix inequality (LMI) was proposed for the uncertain singular SDC system using an instant performance index.

To improve the reliability and security of practical control systems, fault diagnosis (FD) and fault tolerant control (FTC) has long been an important domain of control theory and applications. A number of FD approaches for non-Gaussian singular SDC system have been proposed in recent years [6–9], but few literatures focused on FTC. In literature [8], an iterative learning observer is designed to diagnose the fault in the non-Gaussian singular stochastic distribution system. Based on the estimated fault information, the optimal fault tolerant controller is designed to make the post-fault PDF still track the given distribution. In literature [9], an anti-disturbance fault diagnosis scheme is proposed for the non-Gaussian stochastic distribution control system with multiple disturbances, in which a disturbance observer is designed to estimate and compensate the model disturbance, and H-infinity optimization technology is applied to attenuate the norm bounded disturbance. There are some FD results for general SDC systems with PDF approximation error [10,11], but few research results are about the FD of singular SDC systems with PDF approximation error, let alone the FTC of singular SDC systems.

When fault occurs in a safe-urgent system such as chemical processes, timely fault diagnosis has become a significant issue. However, because of the data processing and transmission, time delay widely exists in practical control systems. The existence of time delay will make FD, FTC and the closed-loop stability analysis more difficult. It is noted that some FD and FTC algorithms have been proposed in time-delayed SDC systems [12–15]. In literature [15], the rational square-root B-spline is used to approach the output PDF of the non-Gaussian time-delayed stochastic distribution control system. A nonlinear neural network observer-based fault diagnosis algorithm was proposed to diagnose the fault and a fault tolerant controller based on PI tracking control scheme is designed to make the post-fault PDF still track the given distribution.

Dynamic link relations between the inputs and the weights exist in the above mentioned SDC systems. However, in practice, some algebraic relations also exist between the input and the weights, leading to a singular state space model between the

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weights vector and the control input. Such systems are called singular stochastic distribution control systems. The final purpose of the grinding processes is to obtain ore pulp with certain fineness distribution by grinding the raw ore, which is a typical stochastic distribution control system. The main control system of the grinding process, such as the amount of ore, amount of water, amount of the second water, sump level, cyclone feed concentration, pressure, rate of flow, have direct influence on the probability density function of fineness distribution. Such control systems can be regarded as the input control variables which control the system output probability density function. It shows that a singular stochastic distribution control model between the probability density function and the multilevel control loops composed of the above mentioned main control systems can be built [5,8,16]. The contribution of this paper is that active fault tolerant control is first realized for the non-Gaussian singular time-delayed stochastic distribution control system with PDF approximation error.

As described above, only a few work was focused on the singular SDC system, and almost no study considered the time delay factor in singular SDC system. This forms the purpose of our work that fault diagnosis and fault tolerant tracking control algorithms for the non-Gaussian time-delayed singular stochastic distribution control system are formulated in this paper. To estimate the fault occurred in the SDC system with PDF approximation error, an adaptive observer is constructed. A fault tolerant tracking controller is designed to let the distribution tracking error at each time instant satisfy a certain upper bound.

The rest of this paper is organized as follows. The model description is given in Section 2. In Section 3, a fault diagnosis algorithm is proposed. A fault tolerant tracking controller is designed in Section 4. Simulation results are included in Section 5. Finally, some concluding remarks are shown in Section 6.

2. Model description

Denote $\gamma(y,u(t))$ as the probability density function of the system output with y being defined on a known bounded interval $[a,b]$, the continuous singular time-delayed SDC system can be expressed as follows:

$$E\dot{x}(t)=Ax(t)+A_dx(t-\tau)+Bu(t)+NF(t) \quad (1)$$

$$V=Dx(t)$$
$$\sqrt{\gamma(y,u(t))}=C(y)V(t)+h(V(t))B_n(y)+e_0(y,t), y\in[a,b] \quad (2)$$

where $x(t)\in R^n$ is the state vector, $V(t)\in R^{n-1}$ is the weight vector, $u(t)\in R^m$ is the control input vector and $F(t)\in R^m$ is the fault vector and τ is the time delay term. $A\in R^{n\times n}, B\in R^{n\times m}, D\in R^{(n-1)\times n}, E\in R^{n\times n}$ and $N\in R^{n\times m}$ are system parameter matrices with $rank(E)=q<n$ (i. e. E is a singular matrix). Eq. (2) represents the static model of the output probability density function (PDF) approximated by the square-root B-spline model. In Eq. (2), it is denoted that

$$C(y)=[\phi_1(y),\phi_2(y),\dots,\phi_{n-1}(y)]$$
$$V=[\omega_1,\omega_2,\dots,\omega_{n-1}]^T(V\neq 0)$$

where $\phi_i(y)(i=1,2,\dots n,\ n\geq 2)$ is the pre-specified basis functions, $\omega_i(i=1,2,\dots n)$ is the approximation weight which is only related to $u(t)$, n is the number of basis functions and $e_0(y,t)$ is the PDF approximation error.

The following lemma and assumptions are used throughout this paper.

Lemma 1 (*Zhou et al.* [5]). *The pair (E,A) is admissible including regular, impulse free and stable if and only if there is a matrix P such that the following equality and inequality hold*

$$E^TP=P^TE\geq 0$$
$$A^TP+P^TA<0$$

Assumption 1. $h(V)$ satisfies the Lipschitz condition, i. e. $\|h(V_1)-h(V_2)\|\leq\|M_h(V_1-V_2)\|$ where M_h is a known matrix.

Assumption 2. The fault occurred in SDC system is bounded, i. e. $\|F\|\leq M_f/2$, $\|\tilde{F}|\leq M_f$, where M_f is a positive constant.

Assumption 3. The approximation error is bounded, i. e. $\|e_0(y,t)\|\leq M_\rho/(b-a)$, where M_ρ is a positive constant.

With the above assumption and lemma, there are invertible matrices L_1,L_2 to make the following equations hold

$$\overline{E}=L_1EL_2=\begin{bmatrix} I^{r\times r} & 0\\ 0 & 0\end{bmatrix},\quad \overline{P}=L_1PL_2^{-T}=\begin{bmatrix}\overline{P}_{11} & \overline{P}_{12}\\ \overline{P}_{21} & \overline{P}_{22}\end{bmatrix}$$
$$\overline{EP}^T=\overline{PE}^T\geq 0$$

where $\overline{P}_{11}=\overline{P}_{11}^T\geq 0,\overline{P}_{21}=0$.

Remark 1. In practice, the limited invariant basis functions and the statistical sampling [1] will lead to an idea PDF approximation that does not exist. In other words, in practice the approximation errors always exist. Therefore, it is much general to consider the approximation error $e_0(y,t)$ in the output SDC systems.

3. Fault diagnosis

When fault occurs, fault diagnosis should be carried out to obtain the information of the fault. For this purpose, the following fault diagnosis observer can be constructed as

$$E\dot{\hat{x}}(t)=A\hat{x}(t)+A_d\hat{x}(t-\tau)+Bu(t)+N\hat{F}(t)+L\varepsilon(t)$$
$$\hat{V}=D\hat{x}(t)\sqrt{\hat{\gamma}(y,u(t))}=C(y)\hat{V}(t)+h\left(\hat{V}(t)\right)B_n(y)$$
$$\dot{\hat{F}}=-\Gamma_1\hat{F}+\Gamma_2\varepsilon(t)$$
$$\varepsilon(t)=\int_a^b\left[\sqrt{\gamma}-\sqrt{\hat{\gamma}}\right]dy$$
$$=\Sigma_1De+\Sigma_2\left[h(V)-h\left(\hat{V}\right)\right]+\rho(t) \quad (3)$$

where $\hat{x}(t)$ is the estimated system state vector, $\varepsilon(t)$ is the residual signal at time t, $e_1(t)=x(t)-\hat{x}(t),\Sigma_1=\int_a^b C(y)\,dy,\Sigma_2=\int_a^b B_n(y)\,dy$, $\rho(t)=\int_a^b e_0(y,t)\,dy$, thus $\|\rho(t)\|\leq M_\rho$, L and $\Gamma_i(i=1,2)$ are gain matrices with appropriate dimensions to be determined later.

Remark 2. $\rho(t)$ is the integral of the PDF approximation error $e_0(y,t)$ on interval $[a,b]$. The purpose of fault diagnosis of the non-Gaussian SDC system with PDF approximation error and time delay term is to still estimate the change of fault accurately when the PDF approximation error and the time delay factor are considered. For the stability analysis of the following observation error dynamic system, the influence of $\rho(t)$ should certainly be considered.

The observation error dynamic system can be formulated as follows:

$$E\dot{e}_1=E\dot{x}-E\dot{\hat{x}}=Ae_1(t)+A_de_1(t-\tau)+N\tilde{F}-L\varepsilon(t)$$
$$=(A-L\Sigma_1D)e_1(t)+A_de_1(t-\tau)+N\tilde{F}-L\Sigma_2\left[h(V)-h\left(\hat{V}\right)\right]-L\rho(t) \quad (4)$$

Let $L_2^{-1}e_1(t)=\left[\xi_1(t)\ \xi_2(t)\right]^T=\xi(t)$ and it can be obtained that

$$\overline{E}\dot{\xi}=L_1(A-L\Sigma_1D)L_2\xi(t)+L_1A_dL_2\xi(t-\tau)+L_1N\tilde{F}-L_1L\Sigma_2\Big[h(V)$$