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Brief Papers

Dissipativity analysis for discrete-time fuzzy neural networks with leakage and time-varying delays ☆

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ABSTRACT

This paper investigates the problem of dissipativity analysis for discrete-time fuzzy neural network with parameter uncertainties based on interval type-2 (IT2) fuzzy model. The parameter uncertainties are handled via the lower and upper membership functions. The original sufficient conditions are presented by a set of linear matrix inequalities (LMIs) to guarantee the dissipativity of the resulting system. The main contribution of this paper is that the discrete-time form of the IT2 T–S fuzzy neural network with leakage and time-varying delays is first proposed. Finally, a numerical example is provided to testify the effectiveness of the proposed results.

1. Introduction

Over the past few years, the neural network has received wide attention and development, such as Hopfield neural network, recursive neural network, bidirectional associative memory neural network, and cellular neural network. In theory, there has been a great amount of progress, for example, computing power of the neural network, the ability to approximate arbitrary continuous mapping and the stability analysis of the dynamic neural network, etc. In applications, neural network has been also rapidly extended to many fields, such as control and optimization, prediction and management [1–3]. Moreover, time delays often exist in neural network model, which is inevitable in practice. The authors in [4] considered unknown time delays in observer-based adaptive neural network nonlinear stochastic control systems. The time-varying delays and external inputs were considered in single inertial BAM neural network in [5]. The authors in [6] considered the problem of global exponential stability for a class of recurrent neural networks with mixed discrete and distributed delays.

Fuzzy model approach is an easy and effective method to handle many complex nonlinear systems [7,8]. In the past few years, the research based on Takagi–Sugeno (T–S) [9] fuzzy-model-based control has aroused considerable concern. The T–S fuzzy model is developed to effectively approximate the nonlinear systems. Many researchers have discussed on T–S fuzzy model to explore its theoretical significance and practical applications [10–13]. The authors studied the H_∞ control performance and stability based on T–S fuzzy model for a nonlinear neural network system in [14]. In [15], the authors studied an improved T–S fuzzy neural network based on declination compensation to satisfy the requirement of the real system. However, it can be found that the above results are on account of type-1 T–S fuzzy neural network-based control systems. Type-1 fuzzy sets are capable of dealing with the nonlinearities of the systems availability but not the uncertainties.

Type-2 fuzzy sets [16–19] are better on handling uncertainties and they can reflect the fuzziness of things well. Therefore, type-2 fuzzy sets have been widely used in various industrial fields, such as fuzzy control [20–22], ecological system [23] and function approximation [24]. Recently, according to lower and upper membership functions scheme, type-2 fuzzy models have been studied in [25–32]. The authors in [33] designed a fuzzy filter for a class of fuzzy-model-based nonlinear networked systems by the IT2 fuzzy model. In [34], the authors considered the tracking control problem for continuous-time T–S fuzzy systems to ensure that the output of the closed-loop system can track the output of a given reference model well. The mean square exponential stability problem of stochastic fuzzy neural networks was investigated in [35]. The authors in [36] studied the dissipativity problem for IT2 stochastic fuzzy neural networks with time-varying delays. Nevertheless, it is worth mentioning that there are few results on

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the dissipativity analysis with leakage and time-varying delays for discrete-time fuzzy neural network with parameter uncertainties based on IT2 fuzzy model. It is a challenge to analyze the dissipativity for discrete-time fuzzy neural network with leakage and time-varying delays based on IT2 fuzzy model.

Accordingly, the problem of dissipativity for the discrete-time form of the IT2 T–S fuzzy neural network with leakage and time-varying delays is discussed in this paper. The main contribution is that the problem of dissipativity analysis for discrete-time fuzzy neural network subject to parameter uncertainties on the basis of IT2 fuzzy model is first discussed. The uncertainties can be well captured by lower and upper membership functions via IT2 T–S fuzzy model approach. First, the definition of strict dissipativity is given for discrete-time fuzzy neural network. Then, novel sufficient conditions are given by a set of linear matrix inequalities (LMIs), which can be easily resolved. Finally, an example is shown to illustrate the availability of the proposed method. The rest of this paper is structured as follows. Section 2 provides the IT2 T–S fuzzy model description and some pre-knowledge of this paper. Section 3 presents the dissipativity conditions for the discrete-time form of the IT2 T–S fuzzy neural network with leakage and time-varying delays. Section 4 shows a simulation example to testify the effectiveness of the proposed results. Finally, Section 5 concludes this paper.

Notations: Throughout the paper, the notations used are fairly standard. "I" denotes an identity matrix with appropriate dimension and "$0_{m\times n}$" denotes $m\times n$ zero matrix. "T" represents the transpose. "$P>0$" means that P is positive definite matrix. $\mathbb{R}^n$ denotes n-dimension Euclidean space. diag$\{\cdots\}$ denotes a block diagonal matrix. "$*$" represents symmetric terms in a block matrix. $l_2[0,\infty)$ means the space of square-integrable vector functions over $[0,\infty)$.

2. Problem formulation

Consider the following discrete-time form of the IT2 T–S fuzzy neural network with ς-rules:

Fuzzy Rule i: IF $u_1(k)$ is U_1^i, …, and $u_\alpha(k)$ is U_α^i, …, and $u_\iota(k)$ is U_ι^i, THEN

$$x(k+1)=-A_ix(k)+A_{di}x(k-\delta)+C_if(x(k))+D_if(x(k-d(k)))+E_iw(k),$$
$$z(k)=B_ix(k)+G_iw(k), \tag{1}$$

where U_α^i stands for the fuzzy set for $i=1,2,\ldots,\varsigma$. $u_\alpha(k)$ is the measurable premise variable for $\alpha=1,2,\ldots,\iota$; ς represents the number of rules and ι represents the number of fuzzy sets. $x(k)=[x_1(k),x_2(k),\ldots,x_n(k)]^T$ denotes the neuron state variable; δ is the leakage delay and $d(k)$ is the time-varying delay satisfying $d_m\le d(k)\le d_M$. $z(k)\in\mathbb{R}^m$ denotes the system output; $w(k)\in\mathbb{R}^l$ is assumed to be a disturbance input belonging to $l_2[0,\infty)$; $f(x(k))=[f_1(x_1(k)),f_2(x_2(k)),\ldots,f_n(x_n(k))]^T\in\mathbb{R}^n$; $f_j(x_j(k))$ is the jth neuron activation function. $A_i=\text{diag}\{a_{1i},a_{2i},\ldots,a_{ni}\}$ is a positive diagonal matrix. A_{di}, B_i, C_i, D_i, E_i, G_i are known real constant matrices. The following interval set stands for the emission intensity of the ith rule:

$$\Phi_i(x(k))=\left[\underline{\phi}_i(x(k)),\overline{\phi}_i(x(k))\right],\quad i=1,2,\ldots,\varsigma,$$

where

$$\underline{\phi}_i(x(k))=\underline{\mu}_{U_1^i}(u_1(k))\times\underline{\mu}_{U_2^i}(u_2(k))\times\cdots\times\underline{\mu}_{U_\iota^i}(u_\iota(k)),$$
$$\overline{\phi}_i(x(k))=\overline{\mu}_{U_1^i}(u_1(k))\times\overline{\mu}_{U_2^i}(u_2(k))\times\cdots\times\overline{\mu}_{U_\iota^i}(u_\iota(k)),$$
$$\overline{\mu}_{U_\alpha^i}(u_\alpha(k))\ge\underline{\mu}_{U_\alpha^i}(u_\alpha(k))\ge 0,\quad \overline{\phi}_i(x(k))\ge\underline{\phi}_i(x(k))\ge 0,$$
$$1\ge\overline{\mu}_{U_\alpha^i}(u_\alpha(k))\ge 0,\quad 1\ge\underline{\mu}_{U_\alpha^i}(u_\alpha(k))\ge 0,$$

$\underline{\phi}_i(x(k))$ denotes the lower grade of membership and $\overline{\phi}_i(x(k))$ denotes the upper grade of membership. $\underline{\mu}_{U_\alpha^i}(u_\alpha(k))$ and $\overline{\mu}_{U_\alpha^i}(u_\alpha(k))$ represent the lower membership function and the upper membership function, respectively. Then system (1) can be written as:

$$x(k+1)=\sum_{i=1}^{\varsigma}\phi_i(x(k))\big[-A_ix(k)+A_{di}x(k-\delta)+C_if(x(k))$$
$$+D_if(x(k-d(k)))+E_iw(k)\big],$$
$$z(k)=\sum_{i=1}^{\varsigma}\phi_i(x(k))[B_ix(k)+G_iw(k)], \tag{2}$$

where for $i=1,2,\ldots,\varsigma$,

$$\phi_i(x(k))=\frac{\underline{\alpha}_i(x(k))\underline{\phi}_i(x(k))+\overline{\alpha}_i(x(k))\overline{\phi}_i(x(k))}{\sum_{\epsilon=1}^{\varsigma}(\underline{\alpha}_\epsilon(x(k))\underline{\phi}_\epsilon(x(k))+\overline{\alpha}_\epsilon(x(k))\overline{\phi}_\epsilon(x(k)))}\ge 0,$$
$$\sum_{i=1}^{\varsigma}\phi_i(x(k))=1,\ \underline{\alpha}_i(x(k))\in[0,1],\quad \overline{\alpha}_i(x(k))\in[0,1],$$
$$1=\underline{\alpha}_i(x(k))+\overline{\alpha}_i(x(k)),$$

$\underline{\alpha}_i(x(k))$ and $\overline{\alpha}_i(x(k))$ depend on parameter uncertainties and are unnecessary to be known in this paper. $\phi_i(x(k))$ is the embedded membership functions.

Definition 1 (*Pan et al. [36]*). The discrete-time fuzzy neural network (2) is called strict (H,S,R)-α-dissipativity, if for any $\alpha>0$ and satisfies

$$J(w,z,T)\ge\alpha<w,w>_T,\quad\forall T>0,$$

under zero initial state, and the energy supply function of discrete-time fuzzy neural network (2) is defined as

$$J(w,z,T)=\langle z,Hz\rangle_T+2<z,Sw>_T+\langle w,Rw\rangle_T,\quad\forall T>0,$$

where H, S and R are real matrices with H and R being symmetric matrices, and $<\varkappa,v>_T=\sum_{k=0}^{T}\varkappa^T(k)v(k)$.

Lemma 1 (*Xu et al. [37]*). *For any constant positive definite matrix M and integers $h_2>h_1$, vector function $\varphi(i)$ satisfies*

$$(h_2-h_1+1)\sum_{i=h_1}^{h_2}\varphi^T(i)M\varphi(i)\ge\left(\sum_{i=h_1}^{h_2}\varphi(i)\right)^TM\left(\sum_{i=h_1}^{h_2}\varphi(i)\right).$$

Assumption 1 (*Liu et al. [6]*). The neural activation functions $f_j(\cdot)$ satisfy

$$f_j^-\le\frac{f_j(\kappa_1)-f_j(\kappa_2)}{\kappa_1-\kappa_2}\le f_j^+,\quad\kappa_1,\kappa_2\in\mathbb{R} \tag{3}$$

for all $\kappa_1\ne\kappa_2$, where f_j^- and f_j^+ are some constants, and

$$F_1=\text{diag}\{f_1^-,f_2^-,\ldots,f_n^-\},\quad F_2=\text{diag}\{f_1^+,f_2^+,\ldots,f_n^+\}.$$

3. Main results

In this section, the dissipativity conditions of the discrete-time IT2 T–S fuzzy neural network (2) are presented in Theorem 1. Based on the LMI approach, we can obtain the following theorem.

Theorem 1. *The discrete-time IT2 T–S fuzzy neural network (2) is mean strict (H,S,R)-α-dissipativity, if there exist parameter matrices $P>0,Q_{u>0}$ $(u=1,\ldots,5),Z_1>0,Z_2>0$, $M=\text{diag}\{m_1,m_2,\ldots,m_n\}\ge 0$ with appropriate dimensions satisfying the following LMIs (i=1,2, …,ς):*

$$\Omega_i=\begin{bmatrix}\Omega_{1i} & \Omega_{2i}\\ \Omega_{3i} & \Omega_{4i}\end{bmatrix}<0,\quad\forall i, \tag{4}$$