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Lag consensus of the second-order leader-following multi-agent systems with nonlinear dynamics

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ABSTRACT

Lag consensus is a phenomenon where followers track the trajectory of the leader with a time delay. By using lag consensus, a protocol is designed for agents to behind the leader at different times, so as to avoid congestion. In this paper, aiming to the lag consensus of second-order nonlinear multi-agent systems, a control protocol for each follower based on local information of neighboring agents is proposed, and an adaptive feedback control protocol is also given. Moreover, the multi-agent systems with noisy environment are considered. The results suggest that our protocol is robust to the noise. Finally, simulation examples are given to illustrate our theoretical analysis.

1. Introduction

Consensus problems, as a basic and fundamental research topic in distributed coordination of multi-agent systems, have drawn a great deal of attention from different researches in recent years, due to its broad range of applications in cooperative control of unmanned air vehicles, formation control of mobile robots and flocking of multiple agents. In the past few years, many significant results of first-order systems have been obtained. Olfati-Saber and Murray presented a systematic framework to analyze the first order consensus algorithms for both fixed and switching topologies, and indicated that the consensus problem can be solved if the diagraph is strongly connected [1]. Ren and Beard pointed out that the interaction topology with a spanning tree is critical for a swarm system to achieve consensus [2]. The research advances of first-order consensus problem refer to the papers [3–5] and the books [6–8].

More recently, the second-order consensus of multi-agent systems have received increasing attention. The second-order multi-agent systems are determined by both position and velocity states, and there is no guarantee of stability, even for strongly connected or spanning-tree graph topologies, if the gains are unconstrained [9]. Therefore, the extension of consensus algorithms from first-order to second-order is non-trivial. In most existing works, each agent can be modeled as a simple linear system. This assumption makes technical analysis much easier and allows using graph theory [10–13]. However, in reality, mobile agents may be governed by more complicated nonlinear dynamics. Indeed, second-order consensus problems with nonlinear agent dynamics have been investigated in networks with fixed topologies [14,15]. On the other hand, much progress has been recently achieved in investigating leader–following consensus, in which the task for all the agents is to follow the leader asymptotically. For example, Ren et al. have discussed the leaderless consensus and the leader-following consensus problem [16]. Meng et al. studied the leader-following consensus problem for a group of agents with identical linear systems subject to control input saturation [17]. Song et al. have investigated the leader-following consensus in a network of agents with nonlinear second-order dynamics via pinning control [18].

It is well known that the information flow in networks is not instantaneous in general, where time delays widely exist. To our knowledge, the main problem involved in consensus with time delay is to study the effects of time delay on the convergence [19–22] and consensusability [13,23,24]. Meanwhile, lag consensus problems of agents are rarely taken into account, except a few papers such as Xie et al. [25]. The lag consensus means that the corresponding state vectors of followers are behind the leader with a time delay. When the time delay is equal to zero, the agents will reach consensus. By using lag consensus, a protocol is designed for agents to behind the leader at different times, so as to avoid congestion. For example, three isolated clusters of vehicles follow the leader and pass across the obstacle, obviously, they cannot pass across the obstacle at the same time (see the left side of Fig. 1). Then, one can design a protocol in such a way that

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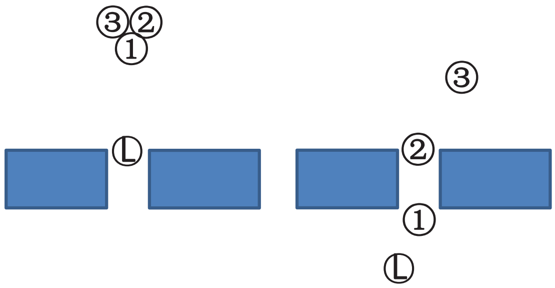

Fig. 1. ⓛ denote the leader, and ⓚ denote the kth cluster of vehicles, $k=1,2,3$.

the ith cluster of vehicles is behind the leader with a time τ_k for $k=1,2,3$ and $0<\tau_1<\tau_2<\tau_3$ (see the right side of Fig. 1). Therefore, it is very meaningful to design a strategy for lag consensus of second-order nonlinear multi-agent system.

Notice furthermore that, as shown in Hong et al. [10] and Ren [16], the velocity states of agents are often unavailable. In this paper, a control protocol for follower based on local position information of neighboring agents is proposed for multi-agent systems to achieve lag consensus, and an adaptive feedback control protocol is given. Moreover, lag consensus for the multi-agent systems with noise environment is considered. All the above fundamental lag consensus criteria are based on Lyapunov functional method, matrix theory, stability theory in stochastic differential equations. Finally, two simulations are given to illustrate the effectiveness of our results.

The organization of the paper is as follows. Some preliminaries are introduced in Section 2 and lag consensus analysis of the second-order multi-agent systems with nonlinear dynamics is discussed in Section 3. Lag consensus of the multi-agent systems with an adaptive feedback control is considered in Section 4. Lag consensus of the multi-agent systems in noisy environment is discussed in Section 5. The paper ends with illustrative examples followed by conclusions.

2. Preliminaries

2.1. Graph theory

Let $G=(\mathcal{V},\mathcal{E})$ be a weighted directed graph of order N, with the set of nodes $\mathcal{V}=\{\mathfrak{v}_1,\mathfrak{v}_2,\ldots,\mathfrak{v}_n\}$, the set of directed edges $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. An edge e_{ij} in graph G is denoted by the ordered pair of vertices, where $\mathfrak{v}_j$ and $\mathfrak{v}_i$ are called the parent and child vertices, respectively, meaning that nodes $\mathfrak{v}_i$ can receive information from node $\mathfrak{v}_j$ [26]. The set of neighbors of vertices $\mathfrak{v}_i$ is denoted by $\mathcal{N}_i=\{j\in\mathcal{V}:(j,i)\in\mathcal{E}\}$.

The following notations are used throughout this paper. Let I_n be the identity matrix of dimension n, $\mathbf{1}_n=[1,\ldots,1]^T\in R^n$, and $\mathbf{0}_n=[0,\ldots,0]^T\in R^n$. $\|\cdot\|$ is the Euclidean norm $\|\cdot\|_2$ in the Euclidean space R^n, $C([-\tau,0];R^d)$ is the space of all continuous R^d-valued functions defined on $[-\tau,0]$ with a norm $\|\varphi\|_\infty=\sup_{-\tau\le\zeta\le 0}\|\varphi(\zeta)\|$, $L^p_{\mathcal{F}_t}([-\tau,0];R^d)$ is the family of all $\mathcal{F}_t$-measurable $C([-\tau,0];R^d)$-valued random variables ϕ such that expectation $E\|\phi\|^p_\infty<\infty$, $\otimes$ denotes the Kronecker product. We say $X>0$ (resp., $X<0$) if the symmetric matrix $X\in R^{n\times n}$ is positive definite (resp., negative definite). Given a symmetric matrix $P\in R^{n\times n}$, we denote $\lambda_{max}(P)$ the maximum of the eigenvalues of P and $\lambda_{min}(P)$ the minimum of the eigenvalues of P. In addition, $diag\{\lambda_1,\ldots,\lambda_n\}$ defines a diagonal matrix with diagonal elements $\lambda_1,\ldots,\lambda_n$.

2.2. Models description

For consensus problems in leader-following nonlinear multi-agent systems, researchers mainly focus on the situations that the states of the leader and followers asymptotically remain identical with time evolution and does not pay attention to the lag consensus phenomenon. For simplicity, we consider a situation where an isolated cluster of agents follows the leader. One can extend the results to deal with some isolated clusters of agents.

The leader $\mathfrak{v}_0$ for multi-agent system is an isolated agent described by

$$\dot{x}_0(t)=v_0(t),\qquad \dot{v}_0(t)=f(x_0(t),v_0(t)),\tag{1}$$

where $x_0(t)\in R^m$ and $v_0(t)\in R^m$ denote the position and velocity of the leader, respectively.

The dynamical behavior of the ith follower $\mathfrak{v}_i$ is in the following form:

$$\dot{x}_i(t)=v_i(t),\qquad \dot{v}_i(t)=f(x_i(t),v_i(t))+u_i(t),\tag{2}$$

where $x_i(t),v_i(t)$ and $u_i(t)\in R^m$ are the position, velocity and control input vector of agent v_i, respectively, and $f(x_i(t),v_i(t))$ is the intrinsic nonlinear dynamics of agent v_i $(i=1,2,\ldots,n)$.

For the system (2), we consider a neighbor-based consensus algorithms as follows:

$$u_i(t)=\alpha\sum_{j\in\mathcal{N}_i}a_{ij}\left[x_j(t)-x_i(t)\right]-b(x_i(t)-x_0(t-\tau))-c(v_i(t)-v_0(t-\tau)),\tag{3}$$

where α is a positive number.

Definition 1. A multi-agent system (2) is said to achieve second-order leader-following lag consensus if, for any initial states, the solutions of (2) satisfy $\lim_{t\to+\infty}\|x_i(t)-x_0(t-\tau)\|=0$ and $\lim_{t\to+\infty}\|v_i(t)-v_0(t-\tau)\|=0$ for a constant $\tau>0$ and all $i=1,2,\ldots,n$.

Let $\hat{x}_i(t)=x_i(t)-x_0(t-\tau)$, $\hat{v}_i(t)=v_i(t)-v_0(t-\tau)$, $\hat{x}(t)=[\hat{x}_1^T(t),\hat{x}_2^T(t),\ldots,\hat{x}_n^T(t)]^T$, $\hat{v}(t)=[\hat{v}_1^T(t),\hat{v}_2^T(t),\ldots,\hat{v}_n^T(t)]^T$, $\hat{\xi}(t)=(\hat{x}^T(t),\hat{v}^T(t))^T$. From (1)–(3), one has

$$\dot{\hat{\xi}}(t)=\begin{bmatrix}0 & I_n\\ \alpha A-bI_n & -cI_n\end{bmatrix}\otimes I_m\hat{\xi}(t)+\begin{bmatrix}0\\ G(t)\end{bmatrix},\tag{4}$$

where $G(t)=F(x(t),v(t))-\mathbf{1_n}\otimes f(x_0(t-\tau),v_0(t-\tau)), a_{ii}=-\sum_{j=1,j\ne i}^{n}a_{ij}(i=1,2,\ldots,n), F(x(t),v(t))=[f^T(x_1(t),v_1(t)),\ldots,f^T(x_n(t),v_n(t))]^T$,

$$A=\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ a_{n1} & a_{n2} & \cdots & a_{nn}\end{bmatrix}.$$

In the following, we give some useful lemmas for further discussion.

Lemma 1 (*Schur complement, Boyd et al. [27]*). *The following linear matrix inequality*

$$\begin{bmatrix}Q(x) & S(x)\\ S^T(x) & R(x)\end{bmatrix}>0,$$

where $Q(x)=Q^T(x), R(x)=R^T(x)$, is equivalent to one of the following conditions:

(1) $Q(x)>0, R(x)-S^T(x)Q^{-1}(x)S(x)>0$;
(2) $R(x)>0, Q(x)-S(x)R^{-1}(x)S^T(x)>0$.

Lemma 2 (*Geršhgorin Circle Theorem, Horn et al. [28]*). *Let a matrix $A=(a_{ij})^{n\times n}$*,

$$R_i(A)=\sum_{j=1,j\ne i}^{n}|a_{ij}|$$