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Existence and exponential stability of solutions of NNs with continuously distributed delays ☆

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ABSTRACT

In this paper, applying the fixed point theorem and a new method, we study the existence, uniqueness, and global exponential stability of solutions of NNs with continuously distributed delays and obtain some new results in terms of system parameters. In our results, the exponentially convergent rate is given and the conditions are less restrictive than previously known criteria. Therefore they can be applied to NNs with a broad range of activation functions though these functions are neither differentiability nor strict monotonicity.

1. Introduction

For neural networks (NNs) with delays, by constructing a suitable Lyapunov function for the system and then to derive sufficient conditions ensuring stability, it is usually not an easy task and these sufficient conditions are wanted to be very restrictive. For the details, see [1–10]. Therefore we need some new methods.

In models with delayed feedbacks, the use of constant discrete delays provides a good approximation to simple circuits consisting of a small number of neurons. But neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths. Thus, there will be a distribution of propagation delays. In this case, the signal propagation is no longer instantaneous and cannot be modeled with discrete time delay. A more appropriate way is to incorporate distributed delays. Moreover, a neural network model with distributed delay is more general than that with discrete delay. This is because the distributed delay becomes a discrete delay when the delay kernel is a δ function at a certain time. For details, see [6–10].

In this paper, we consider the following NNs with continuously distributed delays:

$$x_i'(t)=-\mu_i x_i(t)+\sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j\left(\int_0^\infty k_j(s)x_j(t-s)\,ds\right)+I_i(t). \tag{1.1}$$

where n is the number of units in a neural network, $f_i, I_i \in C(\mathbb{R})$, μ_i, a_{ij}, b_{ij} are constants with $\mu_i>0$, μ_i represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs, a_{ij} is the connection weights between ith unit and jth unit at time t, b_{ij} is the connection weights between ith unit and jth unit, f_j, g_j $(j=1,2,\ldots,n)$ are signal transmission functions, I_i, x_i $(i=1,2,\ldots,n)$ are the activation and external inputs of the ith neuron, respectively, k_j denotes the refractoriness of the jth neuron after it has fired or responded.

$x_i(t)=\varphi_i(t)$ $(i=1,2,\ldots,n.)$ for $t\in(-\infty,\ 0]$ where $\varphi_i(t)\in C([-\infty,\ 0],\ \ \mathbb{R})$.

In the previous exponential stability and existence results of periodic solutions for (1.1), the following condition were used [1–10].

(H_0) $k_j\in C([0,\infty),[0,\infty))$ and

$$\int_0^\infty s k_j(s)\,ds<\infty,\quad j=1,2,\ldots,n;$$

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(H_1) $k_j \in C([0,\infty),[0,\ \infty))$ and there exists $k_j>0$ such that

$$\int_0^\infty k_j(s)\,ds = k_j > 0, \quad j=1,2,\ldots,n;$$

(H_2) there exists $p_j, q_j > 0$ $(j=1,2,\ldots,n)$ such that

$$|f_j(u)-f_j(v)| \le p_j|u-v|, \quad u,v\in\mathbb{R} \text{ and } j=1,2,\ldots,n$$

and

$$|g_j(u)-g_j(v)| \le q_j|u-v|, \quad u,v\in\mathbb{R} \text{ and } j=1,2,\ldots,n;$$

(H_3) there exist $I_i \ge 0$ $(i=1,2,\ldots,n)$ such that

$$I_i \ge \max_{t\ge 0}|I_i(t)|, \quad i=1,2,\ldots,n$$

and (H_4) there exist positive constants $d_1,d_2,\ldots,d_n$ such that one of the following conditions (i)–(vi) holds.

(i) $$\sum_{j=1}^n \left[|a_{ij}|p_j+|b_{ij}|q_jc_j+\frac{1}{d_i}(|a_{ji}|d_jp_i+|b_{ji}|d_jq_ic_j)\right] < 2\mu_i$$

for $i=1,2,\ldots,n$;

(ii) $$\frac{1}{d_i}\sum_{j=1}^n(|a_{ji}|d_jp_i+|b_{ji}|d_jq_ic_j) < \mu_i$$

for $i=1,2,\ldots,n$;

(iii) $$\sum_{j=1}^n(|a_{ij}|p_j+|b_{ij}|q_jc_j) < \mu_i$$

for $i=1,2,\ldots,n$;

(iv) $$\sum_{j=1}^n \left[|a_{ij}|p_j+c_j+\frac{1}{d_i}(|a_{ji}|d_jp_i+|b_{ji}|^2d_jq_i^2c_j)\right] < 2\mu_i$$

for $i=1,2,\ldots,n$;

(v) $$\sum_{j=1}^n \left[|a_{ij}|p_j+|b_{ij}|c_j+\frac{1}{d_i}(|a_{ji}|d_jp_i+|b_{ji}|d_jq_i^2c_j)\right] < 2\mu_i$$

for $i=1,2,\ldots,n$;

(vi) $$\sum_{j=1}^n \left[|a_{ij}|p_j+q_jc_j+\frac{1}{d_i}(|a_{ji}|d_jp_i+|b_{ji}|^2d_jq_ic_j)\right] < 2\mu_i$$

for $i=1,2,\ldots,n$;

(vii) $$\sum_{j=1}^n[|a_{ij}|p_j+|b_{ij}|q_j^2c_j+\frac{1}{d_i}(|a_{ji}|d_jp_i+|b_{ji}|d_jc_j)] < 2\mu_i$$

for $i=1,2,\ldots,n$;

(viii) $$\sum_{j=1}^n \left[|a_{ij}|p_j+q_j^2c_j+\frac{1}{d_i}(|a_{ji}|d_jp_i+|b_{ji}|^2d_jc_j)\right] < 2\mu_i$$

for $i=1,2,\ldots,n$.

The problem is whether (H_0) and the conditions (H_4) are necessary? Our answer is no.

The purpose of this paper is to delete (H_0) and the conditions (H_4) in our new results and consider more general solution, i.e., prove that the following theorems hold.

Theorem 1.1. *Under (H_1), (H_3) and the following condition*

(H_2') there exist $p_j,\ q_j,\ h_j,\ l_j>0$ $(j=1,2,\ldots,n)$ it such that

$$|f_j(u)| \le p_j|u|+h_j, \quad u\in\mathbb{R} \text{ and } j=1,2,\ldots,n$$

and

$$|g_j(u)| \le q_j|u|+l_j, \quad u\in\mathbb{R} \text{ and } j=1,2,\ldots,n,$$

then (1.1) has at least one solution.

Theorem 1.2. *Under (H_1)–(H_3), then (1.1) has a unique solution. Moreover, it is globally exponentially stable.*

The paper is organized as follows:

In Section 2, we prove Theorem 1.1 by a new method.

In Section 3, we prove Theorem 1.2, especially, in the proof of exponential stability, we use a new method but not Lyapunov functions.

In Section 4, to demonstrate the application of our results, we give an example.

2. Existence

In this section, we verify Theorem 1.1.

Set

$$\mathbb{X} = \{x:\ x\in C([0,\ \infty),\ \mathbb{R}^n)\}. \tag{2.1}$$

It follows that $\mathbb{X}$ is a Banach space with the norm

$$\|x\| = \max_{t\ge 0}\|x(t)\| \tag{2.2}$$

where $\|x(t)\| = \left(\sum_{i=1}^n|x_i(t)|^r\right)^{1/r}$.

For each $x\in\mathbb{X}$ with $x(t)=(x_1(t),\ x_2(t),\ldots,x_n(t))^T$, set

$$\mathbf{F}x(t) = (\mathbf{F}_1x(t),\mathbf{F}_2x(t),\ldots,\mathbf{F}_nx(t))^T, \tag{2.3}$$

where

$$\mathbf{F}_ix(t) = x_i(0)e^{-(N+\mu_i)t}+\int_0^t e^{-(N+\mu_i)(t-s)}\left[Nx_i(s)+\sum_{j=1}^n a_{ij}f_j(x_j(s)) + \sum_{j=1}^n b_{ij}g_j\left(\int_0^\infty k_j(h)x_j(s-h)\,dh\right)+I_i(s)\right]ds, \quad i=1,2,\ldots,n \tag{2.4}$$

and $N>0$. Then it is easy to see that $x(t)$ is the fixed point of the operator $\mathbf{F}$ if and only if it is a solution of (1.1).

Proof of Theorem 1.1. Let $N\ge 100\max\{\{\sum_{i=1}^n\ [\sum_{j=1}^n|a_{ij}|p_j]^r\}^{1/r}+\{\sum_{i=1}^n[\sum_{j=1}^n|b_{ij}|q_jk_j]^r\}^{1/r},1\}$ be a constant.

Define a bounded, convex and closed subset $\mathbb{B}$ of $\mathbb{X}$ as follows:

$$\mathbb{B} = \left\{x(t)\in\mathbb{X}:\ \|x(t)\|\le R(t),\ t\in\left[\frac{k-1}{N+\min_{1\le i\le n}\mu_i},\frac{k}{N+\min_{1\le i\le n}\mu_i}\right] \text{ and } k\in\mathbb{N}.\right\} \tag{2.5}$$

where

$$R(t) := \frac{NR}{4}e^{-(1/5)(N+\min_{1\le i\le n}\mu_i-4)\left(t-(k-1)/(N+\min_{1\le i\le n}\mu_i)\right)} \times\left(\frac{N+\min_{1\le i\le n}\mu_i+1}{N}-\frac{N+\min_{1\le i\le n}\mu_i-1}{N}e^{-(8/5)\left(t-(k-1)/(N+\min_{1\le i\le n}\mu_i)\right)}\right) \tag{2.6}$$

and

$$R = \max\left\{20\left(\|x(0)\|+\left\{\sum_{i=1}^n\left[\sum_{j=1}^n|b_{ij}|q_jk_j\right]^r\right\}^{1/r}\|\varphi\| + \left\{\sum_{i=1}^n\left(\sum_{j=1}^n|a_{ij}|h_j\right)^r\right\}^{1/r}. + \left\{\sum_{i=1}^n\left(\sum_{j=1}^n|b_{ij}|l_j\right)^r\right\}^{1/r}+\left\{\sum_{i=1}^n I_i^r\right\}^{1/r}\right),1\right\}. \tag{2.7}$$

Then, from $e^t=\sum_{l=0}^\infty t^l/l!$, $\min\{R'(t),\ [e^{-(1/5)(N+\min_{1\le i\le n}\mu_i-4)t}(1-e^{-(8/5)t})]'\}>0$ for $t\in[0,1/(N+\min_{1\le i\le n}\mu_i)]$, (2.1)–(2.7), Minkowski inequality, (H_1), (H_2') and (H_3), one obtains

$$\|\mathbf{F}x(t)\| = \left\{\sum_{i=1}^n|x_i(0)e^{-(N+\mu_i)t}+\int_0^t e^{-(N+\mu_i)(t-s)}[Nx_i(s)\right.$$