Neurocomputing 171 (2016) 510–514

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Pullback attractor for Cohen–Grossberg neural networks with time-varying delays

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ARTICLE INFO

Article history:
Received 17 April 2014
Received in revised form
7 March 2015
Accepted 27 June 2015
Communicated by W. Lu
Available online 11 July 2015

Keywords:
Cohen–Grossberg neural networks
Delays
Pullback attractor

ABSTRACT

Pullback attractor for Cohen–Grossberg neural networks with time-varying delays is investigated. By using the theory of pullback attractors and Lyapunov-Krasovskii functional, some novel criteria are established to ensure the existence of pullback attractor for Cohen–Grossberg neural networks. Finally, two examples are given to illustrate our theoretical results and indicate that two sets of criteria do not include each other.

1. Introduction

In 1983, Cohen and Grossberg proposed Cohen–Grossberg neural networks [1]. As we know, Cohen–Grossberg neural networks is very general and includes Hopfield neural networks, cellular neural networks and bidirectional associative memory neural networks. For the sake of theoretical interest as well as application considerations, a large number of scholars have studied the dynamical behaviors of Cohen–Grossberg neural networks and obtained considerable results. For example, the stability of the equilibrium point is discussed in [2–16], the existence and uniqueness of periodic solutions are investigated in [17–19], synchronization is discussed in [20,21], boundedness is studied in [22].

Attractor is also one of the foundational dynamical behaviors. The theory of global attractors for autonomous systems as developed by Hale [23] owes much to examples arising in the study of (finite and infinite) retarded functional differential equations [24]. Recently, the theory of pullback attractors has been developed for stochastic and non-autonomous systems in which the trajectories can be unbounded when time increases to infinity, allowing many of the ideas for the autonomous theory to be extended to deal with such examples [25–32]. However, as far as we know, there are few published results on the pullback attractor for Cohen–Grossberg neural networks. Therefore, our main aim of this paper is to establish some sufficient criteria on the existence of pullback attractor for Cohen–Grossberg neural networks.

In this paper, we consider the following Cohen–Grossberg neural networks with time-varying delays

$$\frac{dx(t)}{dt} = d(x(t))[-c(x(t)) + Af(x(t)) + Bg(x(t-\tau(t))) + J], \tag{1.1}$$

where $x=(x_1,\ldots,x_n)^T$ is the state vector, $A=(a_{ij})_{n\times n}$ and $B=(b_{ij})_{n\times n}$ represent the connection weight matrix; $d(x(t)) = diag(d_1(x_1(t)),\ldots,d_n(x_n(t)))$ presents an amplification function, $c(x(t))=(c_1(x_1(t)),\ldots,c_n(x_n(t)))^T$ presents an appropriately behavior function; $J=(J_1,\ldots,J_n)^T$ denotes the external bias; activation functions $f(x(t))=(f_1(x_1(t)),\ldots,f_n(x_n(t)))^T$ and $g(x(t))=(g_1(x_1(t)),\ldots,g_n(x_n(t)))^T$ are continuous; there exists positive constants τ and μ such that the transmission delay $\tau(t)$ satisfies $0\le\tau(t)\le\tau, \dot{\tau}(t)\le\mu<1$.

The content of the paper is as follows. Some preliminaries are in Section 2. Main results are presented in Section 3. Numerical examples are given in Section 4. Finally, conclusions are drawn in Section 5.

2. Preliminaries

First, let us introduce some notation.

Let $\tau>0$ be a given positive number (the delay time) and denote by $\mathcal{L}$ the Banach space $C([-\tau,0];R^n)$ endowed with the norm $\|\xi\| = \sup_{s\in[-\tau,0]}|\xi(s)|$, $|\cdot|$ is the Euclidean norm and $C([-\tau,0];R^n)$ is the space of all continuous R^n-valued functions

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http://dx.doi.org/10.1016/j.neucom.2015.06.069
0925-2312/

defined on $[-\tau,0]$. Denote by x_t the element in $\mathcal{L}$ given by $x_t(s)=x(t+s)$ for all $s\in[-\tau,0]$. $A>0$ (respectively, $A\geq 0$) means that matrix A is symmetric positive definite (respectively, positive semi-definite). A^T denotes the transpose of the matrix A. The norm of matrix A is defined as $|A|=\sqrt{\lambda_{max}(A^TA)}$, $\lambda_{min}(A)$ and $\lambda_{max}(A)$ represents the minimum and maximum eigenvalue of matrix A, respectively.

System (1.1) can be written as

$$\frac{dx(t)}{dt}=F(t,\cdot), \tag{2.1}$$

where the map $F(t,\cdot)$ is defined as

$$F(t,\xi)=d(\xi(0))[-c(\xi(0))+Af(\xi(0))+Bg(\xi(-\tau(t)))+J],\quad \xi\in\mathcal{L}. \tag{2.2}$$

$F:R\times\mathcal{L}\to R^n$ is continuous since $f(x(t))$ and $g(x(t))$ are continuous.

Throughout this paper, we assume the following conditions hold.

(A_1) There exist some constants l_i^-,l_i^+,m_i^- and m_i^+ such that for all $x,y\in R(x\neq y)$,

$$l_i^-\leq\frac{f_i(x)-f_i(y)}{x-y}\leq l_i^+,m_i^-\leq\frac{g_i(x)-g_i(y)}{x-y}\leq m_i^+.$$

(A_2) There exist two matrices $\overline{d}=diag\{\overline{d}_1,\ldots,\overline{d}_n\}>0$ and $\underline{d}=diag\{\underline{d}_1,\ldots,\underline{d}_n\}>0$ such that for all $x\in R, i=1,2,\ldots,n$,

$$0<\underline{d}_i\leq d_i(x)\leq\overline{d}_i.$$

(A_3) There exists two matrices $\delta=diag\{\delta_1,\ldots,\delta_n\}>0$ and $\overline{\delta}=diag\{\overline{\delta}_1,\ldots,\overline{\delta}_n\}>0$ such that for all $x,y\in R, i=1,\ldots,n$,

$$x_i(t)c_i(x_i(t))\geq\delta_i x_i^2(t),\ |c_i(x)-c_i(y)|\leq\overline{\delta}_i|x-y|.$$

Remark 1. Under conditions $(A_1)-(A_3)$, F is a bounded map (i.e., maps bounded sets into bounded sets). In fact, for every $\xi\in D=\{\xi:\|\xi\|\leq r, r>0\}\subset\mathcal{L}$, it follows from (2.2) that

$$\begin{aligned}|F(t,\xi)|&\leq|d(\xi(0))|[|c(\xi(0))|+|A\|f(\xi(0))|+|B\|g(\xi(-\tau(t)))|+|J|]\\
&\leq\max_{1\leq i\leq n}\{\overline{d}_i\}[(|c(0)|+\max_{1\leq i\leq n}\{\overline{\delta}_i\}|\xi(0)|)\\
&\quad+\sqrt{\lambda_{max}(A^TA)}(|f(0)|+\max_{1\leq i\leq n}\{|l_i^+|,|l_i^-|\}|\xi(0)|)\\
&\quad+\sqrt{\lambda_{max}(B^TB)}(|g(0)|+\max_{1\leq i\leq n}\{|m_i^+|,|m_i^-|\}|\xi(-\tau(t))|)+|J|]\\
&\leq\max_{1\leq i\leq n}\{\overline{d}_i\}[(|c(0)|+\max_{1\leq i\leq n}\{\overline{\delta}_i\}r)\\
&\quad+\sqrt{\lambda_{max}(A^TA)}(|f(0)|+\max_{1\leq i\leq n}\{|l_i^+|,|l_i^-|\}r)\\
&\quad+\sqrt{\lambda_{max}(B^TB)}(|g(0)|+\max_{1\leq i\leq n}\{|m_i^+|,|m_i^-|\}r)+|J|].\end{aligned}$$

Remark 2. Condition (A_1) is less conservative than that of in [8], since the constants l_i^-,l_i^+,m_i^- and m_i^+ are allowed to be positive, negative numbers or zeros. It is clear that f and g satisfy Lipschitz condition.

It follows from [23,27,28] that for any $(s,\xi)\in R\times\mathcal{L}$, there exists a solution $x(t;s,\xi)$ for system (2.1). We define a solution operator $\phi(t,s)$ which gives the solution (in $\mathcal{L}$) at time t when $x_s=\xi$, via

$$\phi(t,s)\xi=x_t(\cdot;s,\xi).$$

For the following definitions and result, see [27].

Definition 1. Let ϕ be a process on a complete metric space X. A family of compact sets $\{\mathcal{A}(t)\}_{t\in R}$ is said to be a (global) pullback attractor for ϕ if, for all $s\in R$, it satisfies

$$\phi(t,s)\mathcal{A}(s)=\mathcal{A}(t),\quad \text{for all } t\geq s,$$
$$\lim_{s\to\infty}dist(\phi(t,t-s)D,\mathcal{A}(t))=0,\quad \text{for all bounded subsets } D \text{ of } X.$$

In Definition 1, $dist(A,B)$ is the Hausdorff semidistance between A and B, defined as

$$dist(A,B)=\sup_{a\in A}\inf_{b\in B}d(a,b),\ A,B\subseteq X.$$

Definition 2. $\{B(t)\}_{t\in R}$ is said to be absorbing with respect to the process ϕ if, for all $t\in R$ and all $D\subset X$ bounded, there exists $T_D(t)>0$ such that for all $h>T_D(t), \phi(t,t-h)D\subset B(t)$.

Lemma 1. *Suppose that F and $\phi(t,s)$ map bounded sets into bounded sets, and that there exists a family $\{B(t)\}_{t\in R}$ of bounded absorbing sets for ϕ. Then there exists a pullback attractor $\{\mathcal{A}(t)\}_{t\in R}$ for problem (2.1).*

3. Main results

Theorem 1. *Suppose that there exist some matrices $P=diag\{p_1,\ldots,p_n\}>0, Q_i(i=1,2,3,4)\geq 0, U_i=diag\{u_{i1},\ldots,u_{in}\}\geq 0(i=1,2,3)$ such that*

$$\Sigma=\begin{pmatrix}\Sigma_{11}&0&U_1L_2&U_2M_2&0\\ *&\Sigma_{22}&0&0&U_3M_2\\ *&*&\Sigma_{33}&0&0\\ *&*&*&\Sigma_{44}&0\\ *&*&*&*&\Sigma_{55}\end{pmatrix}<0,$$

where $*$ means the symmetric terms,

$$\begin{aligned}&\Sigma_{11}=3\overline{d}^2P-2P\delta\underline{d}-2U_1L_1-2U_2M_1+Q_1+\tau Q_2,\\
&\Sigma_{22}=-2U_3M_1-(1-\mu)Q_1, \Sigma_{33}=\tau Q_4+A^TPA-2U_1,\\
&\Sigma_{44}=Q_3-2U_2,\quad \overline{\Sigma}_{55}=-2U_3-(1-\mu)Q_3+B^TPB,\\
&L_1=diag\{l_1^-l_1^+,\ldots,l_n^-l_n^+\},\quad L_2=diag\{l_1^-+l_1^+,\ldots,l_n^-+l_n^+\},\\
&M_1=diag\{m_1^-m_1^+,\ldots,m_n^-m_n^+\},\\
&M_2=diag\{m_1^-+m_1^+,\ldots,m_n^-+m_n^+\}.\end{aligned}$$

Then there exists a pullback attractor $\{\mathcal{A}(t)\}_{t\in R}$ for system (2.1).

Proof. From $\Sigma<0$, there exists a sufficient small constant $\lambda>0$ such that

$$\overline{\Sigma}=\begin{pmatrix}\overline{\Sigma}_{11}&0&U_1L_2&U_2M_2&0\\ *&\overline{\Sigma}_{22}&0&0&U_3M_2\\ *&*&\overline{\Sigma}_{33}&0&0\\ *&*&*&\overline{\Sigma}_{44}&0\\ *&*&*&*&\overline{\Sigma}_{55}\end{pmatrix}<0, \tag{3.1}$$

where I denotes identity matrix,

$$\begin{aligned}&\overline{\Sigma}_{11}=\lambda P+2\lambda I+3\overline{d}^2P-2P\delta\underline{d}-2U_1L_1-2U_2M_1+e^{\lambda\tau}Q_1+\tau Q_2,\\
&\overline{\Sigma}_{22}=\lambda I-2U_3M_1-(1-\mu)Q_1,\quad \overline{\Sigma}_{33}=2\lambda I+\tau Q_4+A^TPA-2U_1,\\
&\overline{\Sigma}_{44}=2\lambda I+Q_3e^{\lambda\tau}-2U_2,\quad \overline{\Sigma}_{55}=2\lambda I-2U_3-(1-\mu)Q_3+B^TPB.\end{aligned}$$

Let $x(t)$ be an arbitrary solution with $\|x_{t_0}\|\leq r$. The Lyapunov–Krasovskii functional $V(t)$ is defined as

$$\begin{aligned}V(t)=e^{\lambda t}x^T(t)Px(t)+\int_{t-\tau(t)}^{t}e^{\lambda(s+\tau)}[x^T(s)Q_1x(s)+g^T(x(s))Q_3g(x(s))]\,ds\\
+\int_{t-\tau(t)}^{t}\int_s^t e^{\lambda\theta}[x^T(\theta)Q_2x(\theta)+f^T(x(\theta))Q_4f(x(\theta))]\,d\theta\,ds.\end{aligned} \tag{3.2}$$