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Exponential stability criteria for a neutral type stochastic single neuron system with time-varying delays

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ABSTRACT

In this paper, the issue of global exponential stability for a neutral type single neuron system with stochastic effects is investigated. Based on the linear matrix inequality (LMI) approach together with a novel Lyapunov–Krasovskii functional and stochastic analysis theory, sufficient conditions are derived to ensure that the considered system with time-varying delays is globally exponentially stable. Numerical examples are provided to demonstrate the efficiency and less conservatism of the derived theoretical results.

1. Introduction

Stochastic differential systems play an essential role in many practical problems that arise naturally in the various fields of physics, economics, biology, engineering and so on. It is well-known that a real system is generally affected by external disturbances like stochastic perturbations or parameter uncertainties. The stochastic effects can have the ability to create the undesirable dynamic behaviors such as oscillation, divergence, instability or other poor performance. For example, the noise disturbance extensively occurs in biological networks by reason of environmental uncertainties, which is a foremost source of instability. Thus, the study of stochastic differential equations has been paid significant attention and extensive results have been analyzed in the literature (see [1–3] and the references therein).

In the past few decades, neural networks have been widely investigated and also increased growing identification because of their promising applications in many areas such as associative memory, image processing, pattern recognition, signal processing, and combinatorial optimization [4–7]. These attained applications depend on the stability of equilibrium points of neural networks. Since stability is one of the most essential property of dynamic behaviors. The stability problem for neural networks has attracted a great attention in both theoretical and practical importance. Thus, numerous exciting results on this problem have been reported in recent years [8–11]. However, it is worth pointing that the neural networks modeled in the presence of noise will be more robust and explore more states, which will simplify the learning and variation to the changing demands of a real environment. The noise is a crucial feature of the information processing abilities of the neuron system, and not a mere source of disturbance better suppressed. When the stochastic resonance applied to a single neuron model, it significantly enhances its information processing abilities [12,13]. That is, the main advantage of stochastic resonance to neuron model is once the learning stage is complete from the historical data and a neuron model itself can act as a random process. This kind of neuron model represents the inherent behavior and properties of the stochastic system which is closely related to the natural environment. The Boltzmann machine with threshold activation functions is regarded as a famous stochastic neural network with stochastic activation functions [14].

On the other hand, time delays are frequently occurring in neural networks, which can cause complex dynamic behaviors such as instability and oscillations in the system. The delays are generally time-varying in electronic implementation of analog neural networks, which will be unavoidable owing to the finite switching speed of amplifiers or finite speed of information processing. It has been recognized that the time delays will affect the stability of neural system [16]. Further, it is well known that the variable time delays have more practical significance than the constant delays. Besides, a class of dynamical systems contains some data about the derivative of the past state to further analysis and model the dynamics for such complex neural responses. Such systems have been referred as

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neutral-type systems, in which the systems have both the state delay and the state derivative with delay, the so-called neutral delay. Systems with neutral delay frequently appears in various applications such as partial element equivalent circuits and population ecology [17,18].

The problem of stability criteria for single neuron system with neutral-type has been investigated by many researchers in recent years [19–21]. The stability criteria for neutral differential equations are discussed in [22] with both the state delay and neutral delay as constants. Park and Kwon [23] studied the asymptotic behaviors of solutions of neutral delay differential equations in which the delay dependent condition is expressed in terms of LMI and solved by convex optimization algorithm. Based on the descriptor system approach, the delay-dependent and delay-independent stability analysis has been discussed in [24]. Li [25] has investigated the global exponential stability for neutral delayed differential equations. In [26], the authors examined the asymptotical stability of a class of neutral delay equation by using the LMI technique. The exponential stability of certain neutral differential equations is studied in [27]. Chen and Meng [28] discussed the delay-dependent exponential stability for a class of neutral differential equations with time-varying delays. Liao et al. [29] studied the exponential convergence estimates for a neutral-type single neuron system.

In most of the above-mentioned works, the authors studied the stability criteria for the neutral differential equations with constant or time-varying delays and without stochastic effects. To the best of our knowledge, the exponential stability for a neutral type single neuron system with interval time-varying delays and stochastic effects has not been studied yet in the literature and this fact is the main motivation of our work.

The structure of this paper is arranged as follows. In Section 2, we provide the problem statement and necessary preliminaries. Section 3 is devoted to the proof of global exponential stability for the addressed system by using the Lyapunov–Krasovskii functional method and LMI technique. In Section 4, numerical examples are presented to show the effectiveness of the proposed results. Finally, the conclusion is given in Section 5.

2. Problem description

The following notations are quite standard and will be used throughout this paper. The notation $X \geq Y$(respectively, $X > Y$) denotes the matrix $X - Y$ is positive semi-definite (respectively, positive definite). Here X and Y are symmetric matrices of same dimensions. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (i.e. it contains all P-null sets and is right continuous). The operator $\mathbb{E}$ denotes the mathematical expectation with respect to the given probability measure P. Let $r > 0$ and $\mathcal{C}([-r, 0], \mathcal{R})$ denote the family of continuous functions φ from $[-r, 0]$ to $\mathcal{R}$ with the uniform norm $\|\varphi\| = \sup_{\theta \in [-r,0]} \|\varphi(\theta)\|$. The family of all bounded $\mathcal{F}_0$ measurable, $\mathcal{C}([-r, 0], \mathcal{R})$−valued stochastic variables $\varphi = \{\varphi(\theta) : -r \leq \theta \leq 0\}$ satisfying the norm $|\varphi|^2 = \sup_{\theta \in [-r,0]} \mathbb{E}|\varphi(\theta)|^2 < \infty$ is denoted by $\mathcal{C}^2_{\mathcal{F}_0}([-r, 0], \mathcal{R})$.

In the existing literature, the following deterministic equation governing the single neuron model has been considered in [19–29]:

$$\frac{d}{dt}[x(t) + px(t-\tau)] = -ax(t) + b \tanh x(t-\sigma), \quad t \geq 0, \tag{1}$$

where a, b, τ and σ are positive real numbers and $|p| < 1$. The above equation has been studied with different conditions like $\sigma = \tau, \sigma \geq \tau$ and also with time-varying delays $\tau(t), \sigma(t)$. In that earlier works, it is noted that the stochastic disturbances were ignored. In practice, the single neuron systems can be easily subject to external noises. A neuron model from a stochastic point of view with both multiplicative and additive noise has been studied in [15]. Stochastic resonance in a single neuron model has been theoretically investigated in [12]. So, it is important to consider the stochastic disturbances in single neuron model. Due to this point in this paper, we consider the following single neuron stochastic neutral differential equation with interval time-varying delay:

$$\begin{aligned} d[x(t) + px(t-\tau(t))] = {} & \left[-ax(t) + b \tanh x(t-\sigma(t))\right] dt \\ & + g(t, x(t), x(t-\tau(t)), x(t-\sigma(t)))\, dw(t), \quad t \geq 0, \end{aligned} \tag{2}$$

where a and b are positive real numbers and $|p| < 1$.

The time-varying delays $\tau(t) : [0, \infty) \to [\tau_1, \tau_2]$ and $\sigma(t) : [0, \infty) \to [\sigma_1, \sigma_2]$ are bounded functions satisfying $\tau_1 \leq \tau(t) \leq \tau_2, \sigma_1 \leq \sigma(t) \leq \sigma_2$, where $\tau_1, \tau_2, \sigma_1, \sigma_2$ are some real constants and there exist $\phi_1, \phi_2 \in (0, 1)$ such that $\dot{\tau}(t) \leq \phi_1$, $\dot{\sigma}(t) \leq \phi_2$. $w(t)$ is a scalar Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$. Now, it is assumed that the following initial condition for each solution $x(t)$ of system (2) is given as $x(\theta) = \varphi(\theta)$, $\theta \in [-r, 0]$, where $r = \max\{\tau_2, \sigma_2\}$, $\varphi \in \mathcal{C}^2_{\mathcal{F}_0}([-r, 0], \mathcal{R})$.

Moreover, the function $g(\cdot)$ is locally Lipschitz continuous and satisfies the following linear growth condition:

(A_1) There exist constants $\rho_i > 0, i = 1, 2, 3$, such that

$$g^2(t, x(t), x(t-\tau(t)), x(t-\sigma(t))) \leq \rho_1 x^2(t) + \rho_2 x^2(t-\tau(t)) + \rho_3 x^2(t-\sigma(t)).$$

We now rewrite the system (2) in the following equivalent descriptor form:

$$\begin{aligned} dy(t) &= \left[-ax(t) + b \tanh x(t-\sigma(t))\right] dt + g(t, x(t), x(t-\tau(t)), x(t-\sigma(t)))\, dw(t), \\ 0 &= -y(t) + x(t) + px(t-\tau(t)). \end{aligned}$$

Let $f(t) = -ax(t) + b \tanh x(t-\sigma(t))$ and $g(t) = g(t, x(t), x(t-\tau(t)), x(t-\sigma(t)))$. The above equation can be written as follows:

$$\begin{aligned} dy(t) &= f(t)\, dt + g(t)\, dw(t), \\ 0 &= -y(t) + x(t) + px(t-\tau(t)). \end{aligned}$$

Remark 2.1. The neutral system (2) is considered with white noise. It should be mentioned that noise has direct behavioral concerns, from setting perceptual thresholds to disturbing movement precision. A number of strategies have been adopted to use noise in this manner. Further, the proposed single neuron model can be modeled as velocity-controlled oscillators [12], electronic models like RC circuit and LCR model [15].

Definition 2.2. The equilibrium point of the system (2) is said to be exponentially stable in the mean square if there exist $\alpha > 0$ and $\lambda > 0$ such that for every $\varphi \in \mathcal{C}^2_{\mathcal{F}_0}([-r, 0], \mathcal{R})$,

$$\mathbb{E}|x(t)|^2 \leq \lambda e^{-\alpha t} \sup_{\theta \in [-r,0]} \mathbb{E}|\varphi(\theta)|^2, \quad t \geq 0,$$

where α and λ are known as the decay rate and decay coefficient, respectively.

Lemma 2.3 (*Mao [1]*). *Let $n \in [1, +\infty)$ and $\vartheta \in (0, 1)$. For any two real positive numbers $c, d > 0$, then $(c+d)^n \leq \vartheta^{1-n} c^n + (1-\vartheta)^{1-n} d^n$.*

3. Exponential stability results

In this section, the problem of mean square exponential stability of the equilibrium point of stochastic neutral single neuron system with interval time-varying delays is presented by utilizing the Lyapunov–Krasovskii functional, Itô's differential formula and inequality technique.