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The existence and stability of the anti-periodic solution for delayed Cohen–Grossberg neural networks with impulsive effects

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ABSTRACT

In this paper, we investigate the existence and global exponential stability of the anti-periodic solution for delayed Cohen–Grossberg neural networks with impulsive effects. First, based on the Lyapunov functional theory and by applying inequality technique, we give some new and useful sufficient conditions to ensure existence and exponential stability of the anti-periodic solutions. Then, we present an example with numerical simulations to illustrate our results.

1. Introduction

Since Cohen–Grossberg neural networks (CGNNs) have been first introduced by Cohen and Grossberg in 1983 [1], they have been intensively studied due to their promising potential applications in classification, parallel computation, associative memory and optimization problems. In these applications, the dynamics of networks such as the existence, uniqueness, Hopf bifurcation and global asymptotic stability or global exponential stability of the equilibrium point, periodic and almost periodic solutions for networks plays a key role, see [2–7] and references cited therein.

Time delays unavoidably exist in the implementation of neural networks due to the finite speed of switching and transmission of signals. Besides delay effects, it has been observed that many evolutionary processes, including those related to neural networks, may exhibit impulsive effects. In these evolutionary processes, the solutions of system are not continuous but present jumps which could cause instability in the dynamical systems. Since the existence of delays and impulses is frequently a source of instability, bifurcation and chaos for neural networks, it is important to consider both delays and impulsive effects when investigating the stability of CGNNs, see [8–12].

Over the past decades, the anti-periodic solutions of Hopfield neural networks, recurrent neural networks and cellular neural networks have actively been investigated by a large number of scholars. For details, see [13–15,18] and references therein. In [13], the author considered the existence and exponential stability of the anti-periodic solutions for a class of recurrent neural networks with time-varying delays and continuously distributed delays. In [14], by constructing fundamental function sequences based on the solution of networks, the authors studied the existence of anti-periodic solutions for Hopfield neural networks with impulses. In [15], by establishing impulsive differential inequality and using Krasnoselski's fixed point theorem together with Lyapunov function method, the authors investigated the existence and exponential stability of anti-periodic solution for delayed cellular neural networks with impulsive effects. In [16], by constructing fundamental function sequences based on the solution of networks, the authors studied the existence and exponential stability of anti-periodic solutions for a class of delayed CGNNs. However, till now, there are very few or even no results on the problems of anti-periodic solutions for delayed CGNNs with impulsive effects, while the existence of anti-periodic solutions plays a key role in characterizing the behavior of nonlinear differential equations (see [17]). Thus, it is worth investigating the existence and stability of anti-periodic solutions for CGNNs with both time-varying delays and impulsive effects.

Motivated by the above discussions, in this paper, we are concerned with the existence and the exponential stability of anti-periodic solutions for a class of impulsive CGNNs model with periodic coefficients and time-varying delays. By using analysis technique and constructing suitable Lyapunov function, we establish some simple and useful sufficient conditions on the existence and exponential stability of anti-periodic solutions for CGNNs with impulsive effects.

The rest of the paper organized as follows. In Section 2, the proposed model is presented together with some related definitions. In addition, a preliminary lemma is given. Next section is devoted to investigate the existence and exponential stability of anti-periodic solution of the addressed networks. An illustrative example ends Section 4.

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2. Preliminaries

In this paper, we consider the following impulsive CGNNs model with time-varying delays:

$$\begin{cases} x_i'(t) = a_i(x_i(t))\left[-b_i(t, x_i(t)) + \sum_{j=1}^{n} c_{ij}(t) f_j(x_j(t)) + \sum_{j=1}^{n} d_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) + I_i(t)\right], & t \geq 0,\ t \neq t_k, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k) = \gamma_{ik} x_i(t_k), & k \in N \triangleq \{1, 2, \ldots\}, \end{cases} \quad (1)$$

where n denotes the number of neurons in the network, $x_i(t)$ corresponds to the state of the ith unit at time t, $a_i(x_i(t))$ represents an amplification function, $b_i(t, x_i(t))$ is an appropriate behaved function, $f_j(x_j(t))$ and $g_j(x_j(t-\tau_{ij}(t)))$ denote, respectively, the measures of activation to its incoming potentials of the unit j at time t and $t-\tau_{ij}(t)$, $\tau_{ij}(t)$ corresponds to the transmission delay along the axon of the jth unit and is non-negative function, and $I_i(t)$ denotes the external bias on the ith unit at time t. Concerning coefficients of differential system (1), $c_{ij}(t)$ denotes the synaptic connection weight of the unit j on the unit i at time t, $d_{ij}(t)$ denotes the synaptic connection weight of the unit j on the unit i at time $t-\tau_{ij}(t)$, where $\tau_{ij}(t) > 0,\ i, j = 1, 2, \ldots, n$.

Throughout the paper, we always use $i, j = 1, 2, \ldots, n$, unless otherwise stated. The initial conditions associated with system (1) are given by

$$x_i(s) = \varphi_i(s), \quad s \in [-\tau, 0],$$

where $\tau = \max_{1 \leq i,j \leq n}\{\sup_{t \in R^+} \tau_{ij}(t)\}$, $\varphi_i(s) \in C([-\tau, 0], R)$, which denotes the Banach space of all continuous functions mapping $[-\tau, 0]$ into R with $\infty-$norm defined by

$$\|\varphi\|_\infty = \max_{1 \leq i \leq n}\left\{\sup_{s \in [-\tau, 0]} |\varphi_i(s)|\right\}.$$

A solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ of impulsive system (1) is a piecewise continuous vector function whose components belong to the space

$$PC([-\tau, +\infty), R) = \{\varphi(t) : [-\tau, +\infty) \longrightarrow R \text{ is continuous for } t \neq t_k,\ \varphi(t_k^-),\ \varphi(t_k^+) \in R \text{ and } \varphi(t_k^-) = \varphi(t_k)\}.$$

A function $u(t) \in PC([-\tau, +\infty), R)$ is said to be $\omega-$anti $-$ periodic, if

$$\begin{cases} u(t+\omega) = -u(t), & t \neq t_k, \\ u((t_k+\omega)^+) = -u(t_k^+), & k = 1, 2, \ldots . \end{cases}$$

Let R^n be n-dimensional vector space. For any $u = (u_1, u_1, \ldots, u_n)^T \in R^n$, its norm is defined by

$$\|u\|_1 = \max_{1 \leq i \leq n} |u_i|.$$

In addition, we also formulate the following assumptions:

(H$_1$) $a_i \in C(R, R^+)$ and there exist positive constants $\underline{a_i}$ and $\overline{a}_i$ such that

$$\underline{a_i} \leq a_i(u) \leq \overline{a}_i \quad \text{for all } u \in R.$$

(H$_2$) For each u, $b_i(\cdot, u)$ is continuous, $b(t, 0) \equiv 0$ and there exists a continuous and ω-periodic function $\beta_i(t) > 0$ such that

$$\frac{b_i(t, u) - b_i(t, v)}{u - v} \geq \beta_i(t), \quad u, v \in R, \quad u \neq v.$$

(H$_3$) The activation functions $f_j(u)$, $g_j(u)$ are continuous, bounded and there exist Lipschitz constants $L_j^1, L_j^2 > 0$ such that

$$|f_j(u) - f_j(v)| \leq L_j^1 |u - v|, \quad |g_j(u) - g_j(v)| \leq L_j^2 |u - v|, \quad u, v \in R.$$

(H$_4$) $c_{ij}, d_{ij}, \tau_{ij}, I_i \in C(R, R)$, and there exists a constant $\omega > 0$ such that

$$a_i(-u) = a_i(u), \quad b_i(t+\omega, u) = -b_i(t, -u), \quad c_{ij}(t+\omega) f_j(u) = -c_{ij}(t) f_j(-u), I_i(t+\omega) = -I_i(t), \quad d_{ij}(t+\omega) g_j(u) = -d_{ij}(t) g_j(-u), \quad \tau_{ij}(t+\omega) = \tau_{ij}(t).$$

(H$_5$) $|1 + \gamma_{ik}| \leq 1$ and there exists a positive integer q such that

$$[0, \omega] \cap \{t_k\}_{k \geq 1} = \{t_1, t_2, \ldots, t_q\}, \quad t_{k+q} = t_k + \omega, \quad \gamma_{i(k+q)} = \gamma_{ik}, \quad k \in N.$$

About the jumps, $\{t_k\}_{k \geq 1}$ is a strictly increasing sequence of positive numbers such that $t_{k+1} - t_k \geq \kappa$, for all $k \in N$, $t_k \to \infty$ as $k \to \infty$, where $\kappa > 0$. $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k)$ represents the abrupt change of $x_i(t)$ at impulsive moment t_k.

The following definition is given to obtain our results.

Definition 1. Let $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$ be an anti-periodic solution of differential system (1) with initial value $\varphi^* = (\varphi_1^*(s), \varphi_2^*(s), \ldots, \varphi_n^*(s))^T$. If there exist some constants $\lambda > 0$ and $M > 1$ such that for every solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ of system (1) with any initial value $\varphi = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_n(s))^T$,

$$\|x(t) - x^*(t)\|_1 \leq M \|\varphi - \varphi^*\|_\infty e^{-\lambda t}, \quad \forall t \geq 0,$$

where $\|\varphi - \varphi^*\|_\infty = \sup_{-\tau \leq s \leq 0} \max_{1 \leq i \leq n} |\varphi_i(s) - \varphi_i^*(s)|$. Then, the anti-periodic solution $x^*(t)$ of system (1) is said to be globally exponentially stable.

In addition, in the proof of the main results we shall need the following lemma.

Lemma 1. *Let hypotheses* (H$_1$)–(H$_3$) *and* (H$_5$) *be satisfied and suppose there exist n positive constants* $p_1, p_2, \ldots, p_n$ *such that*

$$-\beta_i(t) p_i + \sum_{j=1}^{n} |c_{ij}(t)| L_j^1 p_j + \sum_{j=1}^{n} |d_{ij}(t)| L_j^2 p_j + D_i(t) < 0, \quad t \in [0, \omega], \quad (2)$$

where

$$D_i(t) = \sum_{j=1}^{n} |c_{ij}(t)||f_j(0)| + \sum_{j=1}^{n} |d_{ij}(t)||g_j(0)| + I_i(t).$$

Then any solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ *of system* (1) *with initial condition*

$$x_i(s) = \varphi_i(s), \quad |\varphi_i(s)| < p_i, \quad t \in [-\tau, 0], \quad i = 1, 2, \ldots, n$$

verifies

$$|x_i(s)| < p_i \quad \text{for all } t > 0, \quad i = 1, 2, \ldots, n. \quad (3)$$

Proof. For any assigned initial condition, hypotheses (H$_1$), (H$_2$) and (H$_3$) guarantee the existence and uniqueness of $x(t)$, the