Neurocomputing 196 (2016) 167–174

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Brief Papers

Fuzzy observer-based output feedback control design of discrete-time nonlinear systems: An extended dimension approach

Qingzhong Gao [a,b,*], Dazhi Wang [a], Yan Zhao [b]

[a] College of Information Science and Engineering, Northeastern University, Shenyang 110004, China
[b] Academy of Automation, Shenyang Institute of Engineering, Shenyang 110136, China

ARTICLE INFO

Article history:
Received 11 August 2015
Received in revised form
28 February 2016
Accepted 1 March 2016
Available online 16 March 2016

Keywords:
Fuzzy logic
Nonlinear systems
Asymptotic stability
Observer-based control

ABSTRACT

Recently, previous results of fuzzy observer-based output feedback control design of discrete-time nonlinear systems have been relaxed by using a multi-samples' method but it is not very mature. This study presents some developments which give a family of feasible solutions to improve the design quality of the recent result. To accomplish this work, an systematic multi-samples' approach that is parameter-dependent on normalized fuzzy weighting functions on optional multi-samples' points is presented in the interest of making good use of more helpful information of the discrete-time nonlinear systems. Furthermore, a new Lyapunov function candidate is given to cooperate with the proposed approach while those redundant terms composed of a set of combinations of the $t+1$ sampled point are removed. As a result of the above efforts, the main defects of the recent result can be overcome and its conservatism is further reduced by an efficient way. In particular, a compromising solution in aspect of not only reducing the conservatism but also suppressing the computational burden is obtained in the special case of $m=1$. Finally, an illustrative example is given to validate the effectiveness of the approach presented in this study.

1. Introduction

Fuzzy logic that is made of fuzzy rules based on linguistic knowledge is one complement to mathematical knowledge in addressing systems whose dynamics are not so well understood and whose models cannot be so conveniently established [1,2]. In 1985, the Takagi–Sugeno (T–S) fuzzy model given in [3] was proposed to cope with the analysis of nonlinear dynamical systems. Today's nonlinear control studies [4–10] are highly based on the T–S fuzzy model. Owing to the usage of the so-called single quadratic Lyapunov function, finding Lyapunov functions is still a hard mission, particularly, for complicated T–S fuzzy systems [11]. A main reason for the difficulty is that the requirement of a monotonically decreasing condition with respect to time for a given single Lyapunov matrix is sometimes too strict [12]. For the sake of relaxing this restriction, different kinds of nonparallel distributed compensation control laws have been proposed in recent years, see [13–17] and the cited literature therein. Because more information of the current-time normalized fuzzy weighting function is involved into the process of control design, the conservatism of control synthesis of T–S fuzzy control systems has been becoming more and more relaxed since the past five years.

Usually, most of the aforementioned research literature are on the basis of the assumption that all the system states are measurable, which is not satisfied for most practical systems. In order to overcome this obstacle, a lot of related research works, for instance, filter designs based on the T–S fuzzy model [18–23], different fuzzy observer designs [24–27], static output feedback control of T–S fuzzy systems [28], dynamic output feedback control of T–S fuzzy systems [29], fuzzy observer-based output feedback control designs [30,31], have been developed in recent years. Particularly, the obtained design conditions in [30] were presented for calculating a family of observer-based controllers by means of bilinear matrix inequalities which were not convex and NP-hard to be implemented. As an alternative, two-step procedure was often used but much conservatism was inevitably introduced. Recently, a single-step linear matrix inequality method has been presented in [31]. Using the result given in [31], both the controller and observer parameters have been provided by means of a family of strict linear matrix inequalities, which are numerically feasible with the help of commercially available software. It is noteworthy that the recent approach proposed in [31] is not very mature as it can be summarized by two main reasons as follows: (A) Despite announcing that a multi-samples' method is applied, there are only three sampled points involved in the result of [31], i.e.,

* Corresponding author at: College of Information Science and Engineering, Northeastern University, Shenyang 110004, China.
E-mail address: qingzhonggaoneu@163.com (Q. Gao).

http://dx.doi.org/10.1016/j.neucom.2016.03.004
0925-2312/

$t-1, t, t+1$, respectively. Actually, the problem of developing a systematic multi-samples' approach (the one that can be parameter-dependent on normalized fuzzy weighting functions on optional multi-samples' points) is still unsolved. In general words, the information of multi-samples' normalized fuzzy weighting functions has not been fully used in [31]. (B) Since more information about normalized fuzzy weighting functions on the above three sampled points is considered in [31], the obtained result has become less conservative but some redundant matrix-valued terms have also introduced which always play a negative role in the design process (as a result, the computational burden increases rapidly). Considering this fact, it is very important to find a way to remove these redundant terms. Now, the problem of finding an efficient way to resolve the above two problems needs to be further investigated, which motivates us to actualize the study.

Inspired by the aforementioned concerns, the aim of this study is to investigate the issue of relaxed fuzzy observer-based output feedback controller designs by developing an extended dimension approach, which gives a compromising solution to overcome two main defects of the recent result [31]. To accomplish this work, an systematic multi-samples' approach that is parameter-dependent on normalized fuzzy weighting functions on optional multi-samples' points is presented and therefore more helpful information of the discrete-time nonlinear systems can be integrated into control design. In other words, the problem of finding an efficient way to resolve the first problem in [31] is successfully resolved in the study. Furthermore, a new Lyapunov function candidate is given to cooperate with the proposed approach while those redundant terms composed of a set of combinations of the $t+1$ sampled point are removed in what means the second problem in [31] is also resolved. As a result of the above efforts, the two main defects of the recent result are overcome and the conservatism can be further reduced by an efficient way. In particular, a compromising solution in aspect of not only reducing the conservatism but also suppressing the computational burden is obtained in the special case of $m=1$.

Notations: Throughout this study, an asterisk ($*$) located inside a matrix stands for the transpose of its symmetric term; $\mathbb{Z}_+$ represents the set of positive integers; $\mathbb{R}$ represents the set of real numbers; $p!$ denotes factorial, i.e., $n_1! = n_1(n_1-1)\cdots(1)$ for $n_1 \in \mathbb{Z}_+$ with $0! = 1$; For one matrix, $\mathrm{He}(M)$ is defined as $\mathrm{He}(M) = M + M^T$. Furthermore, the left-hand side of a relation is defined as $\mathrm{Left}(\cdot)$.

2. Preliminaries

A discrete-time complex nonlinear dynamical system is approximated by the following T–S fuzzy model [31]:

$$\begin{cases} x(t+1) = A_{z(t)}x(t) + B_{z(t)}u(t)) \\ y(t) = C_{z(t)}x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_1}$ denotes the system state vector; $u(t) \in \mathbb{R}^{n_2}$ denotes the control input vector; $y(t) \in \mathbb{R}^{n_3}$ denotes the measurement output vector; $A_{z(t)} = \sum_{i=1}^r h_i(z(t))A_i$, $B_{z(t)} = \sum_{i=1}^r h_i(z(t))B_i$, $C_{z(t)} = \sum_{i=1}^r h_i(z(t))C_i$, A_i, B_i and C_i are system matrices; $h_i(z(t))$ is the i-th normalized fuzzy weighting function.

Recently, an observer-based controller has been proposed in [31] and its form is represented as follows:

$$\begin{cases} \hat{x}(t+1) = A_{z(t)}\hat{x}(t) + B_{z(t)}u(t) + G^{-1}_{z(t-1)z(t)}K_{z(t-1)z(t)}(y(t) - \hat{y}(t)), \\ \hat{y}(t) = C_{z(t)}\hat{x}(t), \\ u(t) = \alpha^{-1}_{z(t-1)z(t)}F_{z(t-1)z(t)}\hat{x}(t), \end{cases} \tag{2}$$

where $K_{z(t-1)z(t)}$, $G_{z(t-1)z(t)}$, $F_{z(t-1)z(t)}$ and $\alpha_{z(t-1)z(t)}$ are gain matrices to be solved, and their forms are defined as follows:

$$\begin{aligned} K_{z(t-1)z(t)} &= \sum_{1 \le i \le r, 1 \le j \le r} h_i(t-1)h_jK_{ij}, G_{z(t-1)z(t)} \\ &= \sum_{1 \le i \le r, 1 \le j \le r} h_i(t-1)h_jG_{ij}, F_{z(t-1)z(t)} \\ &= \sum_{1 \le i \le r, 1 \le j \le r} h_i(t-1)h_jF_{ij}, \alpha_{z(t-1)z(t)} = \sum_{1 \le i \le r, 1 \le j \le r} h_i(t-1)h_j\alpha_{ij}, \end{aligned}$$

K_{ij}, G_{ij} and F_{ij} are appropriately dimensional matrices, α_{ij} is one scalar.

Recalling (1) and (2), the closed-loop system is derived as follows:

$$\tilde{x}(t+1) = \begin{pmatrix} \Gamma_{11} & -\alpha^{-1}_{z(t-1)z(t)}B_{z(t)}F_{z(t-1)z(t)} \\ 0 & A_{z(t)} - G^{-1}_{z(t-1)z(t)}K_{z(t-1)z(t)}C_{z(t)} \end{pmatrix}\tilde{x}(t), \tag{3}$$

where $\tilde{x}(t) = \begin{pmatrix} x(t) \\ x(t)-\hat{x}(t) \end{pmatrix}$ and $\Gamma_{11} = A_{z(t)} + \alpha^{-1}_{z(t-1)z(t)}B_{z(t)}F_{z(t-1)z(t)}$.

Then, a set of necessary definitions about homogeneous matrix polynomials are borrowed from the literature (for instance, [15,16,31]). Such as, the set Δ_r is defined as $\Delta_r = \{\alpha \in \mathbb{R}^r; \sum_{i=1}^r \alpha_i = 1; \alpha \ge 0\}$. Based on this, one defines $\alpha_1^{k_1}\alpha_2^{k_2}\cdots\alpha_r^{k_r}, \alpha \in \Delta_r, k_i \in \mathbb{Z}_+, i = 1, 2, \ldots, r$ as the monomials, $k = k_1k_2\cdots k_r$, and $P_k \in \mathbb{R}^{n \times n}, \forall k \in \mathcal{K}(g)$ are matrix-valued coefficients. $\mathcal{K}(g)$ denotes the set of r-tuples derived as all possible combinations of nonnegative integers $k_i, i = 1, 2, \ldots, r$, such that $k_1 + k_2 + \cdots + k_r = g$. As a simple example, for homogeneous polynomials of degree $g=4$ with $r=2$ variables, the possible values of the partial degrees are $\mathcal{K}(3) = \{40, 31, 22, 13, 04\}$. For r-tuples k and k', one denotes $k \ge k'$ if $k_i \ge k'_i, (i = 1, \ldots, r)$. The usual operations of summation, $k + k'$, and subtraction, $k - k'$ (whenever $k \ge k'$), are defined componentwise.

Particularly, two definitions of the r-tuple $\chi_i \in \mathcal{K}(1)$ and the coefficient $\pi(k)$ are defined as [31]:

$$\chi_i = \underbrace{0}_{1-\text{th element}} \cdots 0 \underbrace{1}_{i-\text{th element}} 0\cdots \underbrace{0}_{r-\text{th element}},$$

$$\pi(k) = (k_1!) \times (k_2!) \times \cdots \times (k_r!), \quad \forall k \in \mathcal{K}(g), g \in \mathbb{Z}_+. \tag{4}$$

On the basis of both (3) and (4), a set of LMI-based design conditions have been proposed in [31] as follows.

Proposition 1 (*Ma and Xie [31]*). *The closed-loop system (3) is asymptotically stable, if there exist matrices $F_{ij} \in \mathbb{R}^{n_2 \times n_1}$ and $K_{ij} \in \mathbb{R}^{n_1 \times n_3}$; matrices $G_{ij} \in \mathbb{R}^{n_1 \times n_1}$; symmetric matrices $P_{1ij} \in \mathbb{R}^{n_1 \times n_1}$ and $P_{2ij} \in \mathbb{R}^{n_1 \times n_1}$ and scalars α_{ij}, where $1 \le i \le r, 1 \le j \le r$; such that the LMIs of (5) are satisfied*:

$$\Upsilon_{pkk'} = \begin{bmatrix} \Upsilon^{11}_{pkk'} & * & * & * \\ 0 & \Upsilon^{22}_{pkk'} & * & * \\ \Upsilon^{31}_{pkk'} & \Upsilon^{32}_{pkk'} & \Upsilon^{33}_{pkk'} & * \\ 0 & \Upsilon^{42}_{pkk'} & 0 & \Upsilon^{44}_{pkk'} \end{bmatrix} > 0,$$

$$\forall p \in \mathcal{K}(d_1+1), \quad i \in \{1, 2, \ldots, m\}; k \in \mathcal{K}(d_3+2); k' \in \mathcal{K}(d_2+1); \tag{5}$$

where $d_1, d_2, d_3 \in \mathbb{Z}_+$, and

$$\Upsilon^{11}_{pkk'} = \sum_{\substack{1 \le i \le r, p \ge \chi_i; \\ 1 \le j \le r, k \ge \chi_j}} \left\{ \frac{(d_1)!}{\pi(p-\chi_i)} \frac{(d_3+1)!}{\pi(k-\chi_j)} \frac{(d_2+1)!}{\pi(k')} P_{1ij} \right\},$$

$$\Upsilon^{22}_{pkk'} = \sum_{\substack{1 \le i \le r, p \ge \chi_i; \\ 1 \le j \le r, k \ge \chi_j}} \left\{ \frac{(d_1)!}{\pi(p-\chi_i)} \frac{(d_3+1)!}{\pi(k-\chi_j)} \frac{(d_2+1)!}{\pi(k')} P_{2ij} \right\},$$

$$\Upsilon^{33}_{pkk'} = \sum_{\substack{1 \le i \le r, p \ge \chi_i; \\ 1 \le j \le r, k \ge \chi_j}} \left\{ \frac{(d_1)!}{\pi(p-\chi_i)} \frac{(d_3+1)!}{\pi(k-\chi_j)} \frac{(d_2+1)!}{\pi(k')} 2\alpha_{ij}I \right\}$$